## Spatial Filtering, Model Uncertainty and the Speed of Income Convergence in Europe

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#### STRUCTURE OF THE PRESENTATION

- Our research question: Evaluating robust effects of covariates on regional growth under model uncertainty in terms of
  - the choice of explanatory variables
  - the choice of a spatial weight matrix
- In particular, we are interested in obtaining robust estimates of the proper speed of income convergence (free of spatial spillovers) in Europe and thus quantifying the role of spatial spillovers in European regional growth

#### Structure:

- Model uncertainty
  - Spatial autocorrelation and model uncertainty
  - BMA, spatial weight uncertainty and spatial filtering
- Growth and convergence in EU regions
  - The determinants of regional growth under model uncertainty
  - The speed of income convergence under model uncertainty
- Conclusions



#### The problem of model uncertainty

► A stereotype SAR specification:

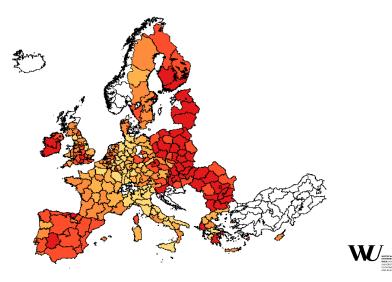
$$y = \alpha + \rho \mathbf{W} y + \mathbf{X}_k \vec{\chi}_k + \sigma \varepsilon,$$

where  $y_i$  refers to regional growth and  $\mathbf{X}_k = x_1, \ldots, x_k$  are k variables which belong to the set  $\mathbf{X}$  of possible determinants of y and  $\varepsilon$  is an error term

- ► How important is a variable  $x_j$  in explaining y if we do not know which model is the true model: robustness of growth determinants
- Estimating the speed of convergence (purged of spatial feedback)
- Bayesian Model Averaging (BMA) presents a fully-fledged systematic approach to dealing with model uncertainty
- ► How can we integrate uncertainty about W in the BMA framework?



# ECONOMIC GROWTH IN EUROPEAN REGIONS 1995-2005



#### BAYESIAN MODEL AVERAGING

Assume

$$y = \alpha + \rho \mathbf{W}_f y + \sum_{k=1}^n \beta_k x_k + \sigma \varepsilon,$$

and a set of competing models,  $\{M_1, \ldots, M_M\}$  defined by the choice of variables in X and a spatial weight matrix  $\mathbf{W}_f$ .

• Our quantity of interest is the effect of variable  $x_j$ ,  $\beta_j$ 

$$\mathbf{P}(\beta_j | \mathbf{Y}) = \sum_{m=1}^{M} \mathbf{P}(\beta_j | \mathbf{Y}, M_m) \mathbf{P}(M_m | \mathbf{Y}),$$

where  $\mathrm{P}(M_k|\mathbf{Y})$  are the posterior model probabilities,

$$P(M_k|\mathbf{Y}) = \frac{P(\mathbf{Y}|M_k)P(M_k)}{\sum_{m=1}^{M} P(\mathbf{Y}|M_m)P(M_m)}$$



### BAYESIAN MODEL AVERAGING

The Bayes factor can be approximated as

$$\frac{\mathrm{P}(\mathbf{Y}|M_2)}{\mathrm{P}(\mathbf{Y}|M_1)} = N^{(k_1-k_2)/2} \left(\frac{Lik_2}{Lik_1}\right),$$

- $\blacktriangleright$  Using  $\mathrm{P}(M_k|\mathbf{Y})\;\forall k$  we can compute  $\mathrm{P}(\beta_j|\mathbf{Y})$
- ► We can also obtain the posterior inclusion probability (PIP) of each variable as the sum of the probabilities of models including it
- The cardinality of the model space makes the computation of all posteriors intractable: MC<sup>3</sup> methods
- The problem is enlarged by the estimation of each individual SAR model



INTROD

### Spatial filtering

- The spatial link matrix  $\mathbf{W}$  is first transformed to satisfy symmetry and then entered in a quadratic form with the projector  $M_1 = I - \iota_N (\iota'_N \iota_N)^{-1} \iota'_N$
- ► The eigenvectors *e* extracted from [M<sub>1</sub><sup>1</sup>/<sub>2</sub>(W + W')M<sub>1</sub>] reflect spatial patterns and can be thought of as proxy variables for the spatial structure of the data
- A linear combination of a reasonable subset of the eigenvectors of the projection matrix is capable of proxying the omitted variables that tie the residuals spatially together or to approximate the spatial process in general
- Spatially filtered specification:

$$y = \alpha + \sum_{i=1}^{E} \gamma_i \vec{e_i} + \mathbf{X}_k \vec{\chi}_k + \sigma \varepsilon,$$





- Start with a model as defined by a group of regressors and the set of eigenvalues associated to a spatial weighting matrix W<sup>k</sup>
- ► First step: A candidate regressor is drawn from the set of potential covariates and we add/drop the candidate regressor to/from the current model. The candidate model, M<sup>k</sup><sub>c</sub> is accepted with probability:

$$\tilde{p}_{cj} = \min\left[1, \frac{\overline{p}(M_c^k)p(y|M_c^k, \theta_c, \mathbf{W}_k)}{\overline{p}(M_j^k)p(y|M_j^k, \theta_j, \mathbf{W}_k)}\right]$$

Second step: Draw a candidate weighting matrix W<sup>c</sup>. The accepted model from step 1) is then compared with the model containing the same regressors and the eigenvalues corresponding to W<sub>c</sub>,

$$\hat{p}_{if} = \min\left[1, \frac{p(y|M_i^c, \theta_i, \mathbf{W}^c)}{p(y|M_f^j, \theta_f, \mathbf{W}^j)}\right].$$

 Repeat a large number of times and compute statistics based on the visited models

#### IS THERE LIFE ON MARS?

$$y = 0.6\mathbf{W}_{j}y + 1.5\mathbf{x}_{1} + 2\mathbf{x}_{4} - 0.5\mathbf{x}_{10} + 0.5\varepsilon,$$

#### We simulate 10 potential covariates

- 4 settings for W:
  - ▶ case 1:  $\mathbf{W}_j$  is a first order Queen contiguity matrix  $(\mathbf{W}_1^Q)$ ,
  - ► case 2:  $\mathbf{W}_{j}$  is a four nearest neighbour weight matrix  $(\mathbf{W}_{4}^{K-NN})$ ,
  - ▶ case 3:  $\mathbf{W}_{j}$  is a 400 km distance band weight matrix  $(\mathbf{W}_{400}^{b})$ , ▶ case 4:  $\mathbf{W}_{j}$  is given by  $0.5\mathbf{W}_{1}^{Q} + 0\mathbf{W}_{4}^{K-NN} + 0.5\mathbf{W}_{400}^{b}$ .



#### A SIMULATION

	$\mathbf{W}_1^Q$	$\mathbf{W}_{4}^{K-NN}$	$\mathbf{W}^{b}_{400}$
Case $j = 1$			
Percentage visited	99.66	0.34	0.00
Adj. $R^2$	0.47	0.37	0.23
# eigenvalues	25.50	23.46	9.02
Case $j = 2$			
Percentage visited	0.00	100.00	0.00
Adj. $R^2$	0.29	0.42	0.18
# eigenvalues	16.98	33.82	7.18
Case $j = 3$			
Percentage visited	0.00	0.00	100.00
Adj. $R^2$	0.09	0.12	0.19
# eigenvalues	2.56	6.56	10.44
Case $j = 4$			
Percentage visited	32.96	16.89	50.15
Adj. $R^2$	0.25	0.22	0.21
# eigenvalues	11.84	13.50	9.66



INTRODUCTION MODEL UNCERTAINTY EMPIRICAL RESULTING CONCL OCOCOOCOCO EMPIRICAL APPLICATION: ECONOMIC GROWTH IN EUROPEAN REGIONS

- ▶ Dependent variable: Growth rate of GDP per capita 1995-2005.
- ► Data for 255 NUTS-2 regions belonging to EU-27
- ▶ 50 potential covariates, evaluated in 1995
- ▶ 16 potential spatial weighting matrices



Empirical results

#### VARIABLES

Variable name	Description	Source		
Factor accumula	tion/convergence			
GDPCAP0	Initial real GDP per capita (in logs)	Eurostat		
gPOP	Growth rate of population	Eurostat		
shGFCF	Share of GFCF in GVA	Cambridge Econometrics		
Infrastructure				
INTF	Proportion of firms with own	ESPON		
	website regression			
TELH	A typology of levels of household	ESPON		
	telecommunications uptake			
TELF	A typology of estimated levels of	ESPON		
	business telecommunications access and uptake			
Seaports	Regions with seaports	ESPON		
AirportDens	Airport density	ESPON		
RoadDens	Road density	ESPON		
RailDens	Rail density	ESPON		
ConnectAir	Connectivity to commercial airports by car	ESPON		
ConnectSea	Connectivity to commercial seaports by car	ESPON		
AccessAir	Potential accessibility air	ESPON		
AccessRoad	Potential accessibility road	ESPON		
Socio-geographic	cal variables			
Settl	Settlement structure	ESPON		
OUTDENS0	Initial output density			
EMPDENS0	Initial employment density			
POPDENS0	Initial population density			
RegCoast	Coast	ESPON		
RegBorder	Border	ESPON		
RegPent27	Pentagon EU 27 plus 2	ESPON		
RegObj1	Objective 1 regions	ESPON		
Capital	Capital city			
Airports	Number of airports	ESPON		
Temp	Extreme temperatures	ESPON		
Hazard	Sum of all weighted hazard values	ESPON		
Distde71	Distance to Frankfurt			
DistCap	Distance to capital city			

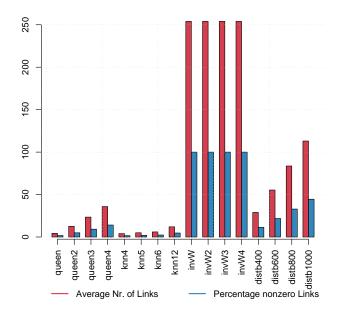


#### VARIABLES

Variable name	Description	Source				
Technological innovation						
PatentT	Number of patents total	Eurostat				
PatentHT	Number of patents in high technology	Eurostat				
PatentICT	Number of patents in ICT	Eurostat				
PatentBIO	Number of patents in biotechnology	Eurostat				
PatentShHT	Share of patents in high technology	Eurostat				
PatentShICT	Share of patents in ICT	Eurostat				
PatentShBIO	Share of patents in biotechnology	Eurostat				
HRSTcore	Human resources in science and technology (core)	Eurostat LFS				
Human capital						
ShSH	Share of high educated in working age population	Eurostat LFS				
ShSL	Share of low educated in working age population	Eurostat LFS				
ShLLL	Life long learning	Eurostat LFS				
Sectoral structure	re/employment					
ShAB0	Initial share of NACE A and B	Eurostat				
	(Agriculture)					
ShCE0	Initial share of NACE C to E	Eurostat				
	(Mining, Manufacturing and Energy)					
EREH0	Employment rate - high	Eurostat LFS				
EREL0	Employment rate - low	Eurostat LFS				
ERET0	Employment rate - total	Eurostat LFS				
URH0	Unemployment rate - high	Eurostat LFS				
URL0	Unemployment rate - low	Eurostat LFS				
URT0	Unemployment rate - total	Eurostat LFS				
ARH0	Activity rate high	Eurostat LFS				
ARL0	Activity rate low	Eurostat LFS				
ART0	Activity rate total	Eurostat LFS				

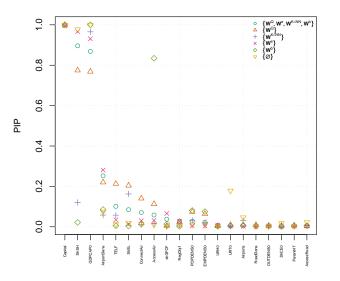


#### Spatial weighting matrices



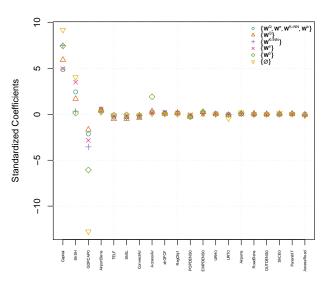


#### POSTERIOR INCLUSION PROBABILITIES





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Empirical results

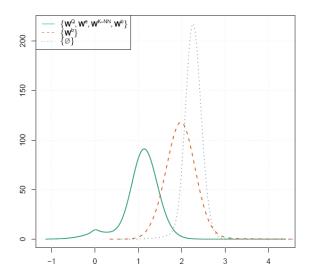
Conclusions 00

#### POSTERIOR INCLUSION PROBABILITIES

	$W^Q_1$	$W^Q_2$	$W^Q_3$	$W^Q_4$	$\mathbf{W}_{4}^{K-NN}$	$\mathbf{W}_{5}^{K-NN}$	$W_6^{K-NN}$	$\mathbf{W}_{12}^{K-NN}$
PIP	0.0234	0.0000	1.7728	36.2936	0.0000	0.0016	0.0000	0.0000
	$W_1^e$	$W_2^e$	$W_3^e$	$W_4^e$	$W_{400}^{b}$	$W_{600}^{b}$	$W^{b}_{800}$	$W^{b}_{1000}$
PIP	0.0000	0.0000	61.8796	0.0000	0.0000	0.0290	0.0000	0.0000



## POSTERIOR DISTRIBUTION ON THE SPEED OF CONVERGENCE





#### CONCLUSIONS

- We put forward a Bayesian Model Averaging method for dealing with model uncertainty in the presence of potential spatial autocorrelation of unknown form.
- We propose using spatial filtering methods to achieve computational gains in the procedure.
- Evaluating growth determinants across European regions for the period 1995-2005, the choice of a particular class of spatial weighting matrices can have an important effect on the estimates of the parameters attached to the model covariates. Our posterior results emphasize the importance of human capital and convergence for economic growth in Europe at the regional level.



#### CONCLUSIONS

- Estimates of the speed of income convergence across European regions depend strongly on the form of the spatial patterns which are assumed to underly the dataset.
- The posterior distribution of the speed of convergence parameter has some probability mass around no convergence (0% speed of convergence) and a clear mode at a rate of convergence of 1%, approximately half of the value which is usually reported in the literature.

