Optimal Pollution, Optimal Population, and Sustainability

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Sustainable?

Traditional, population constant

- Scarcity of natural recources leads to ever-decreasing per capita consumption (Dasgupta and Heal 1974, Krautkraemer 1985, Pezzey and Withagen 1998).
- An optimal path is sustainable if it provides non-decreasing consumption for a non-decreasing population (Brundtland Comission 1987).

Endogenous population

- It may well be optimal to allow some environmental deaths.
- Optimal trend in population may lead to extinction (in infinite time)
- There is not much reason to believe that that people care

Malthus-Hardin model

- Environment is a public good
- Externalities in population growth
- Incentives for births are private (Malthus -> Hardin)
- Carrying capacity may be exceed
- Positive check (rise in mortality) restores the balance
- Tragedy of commons

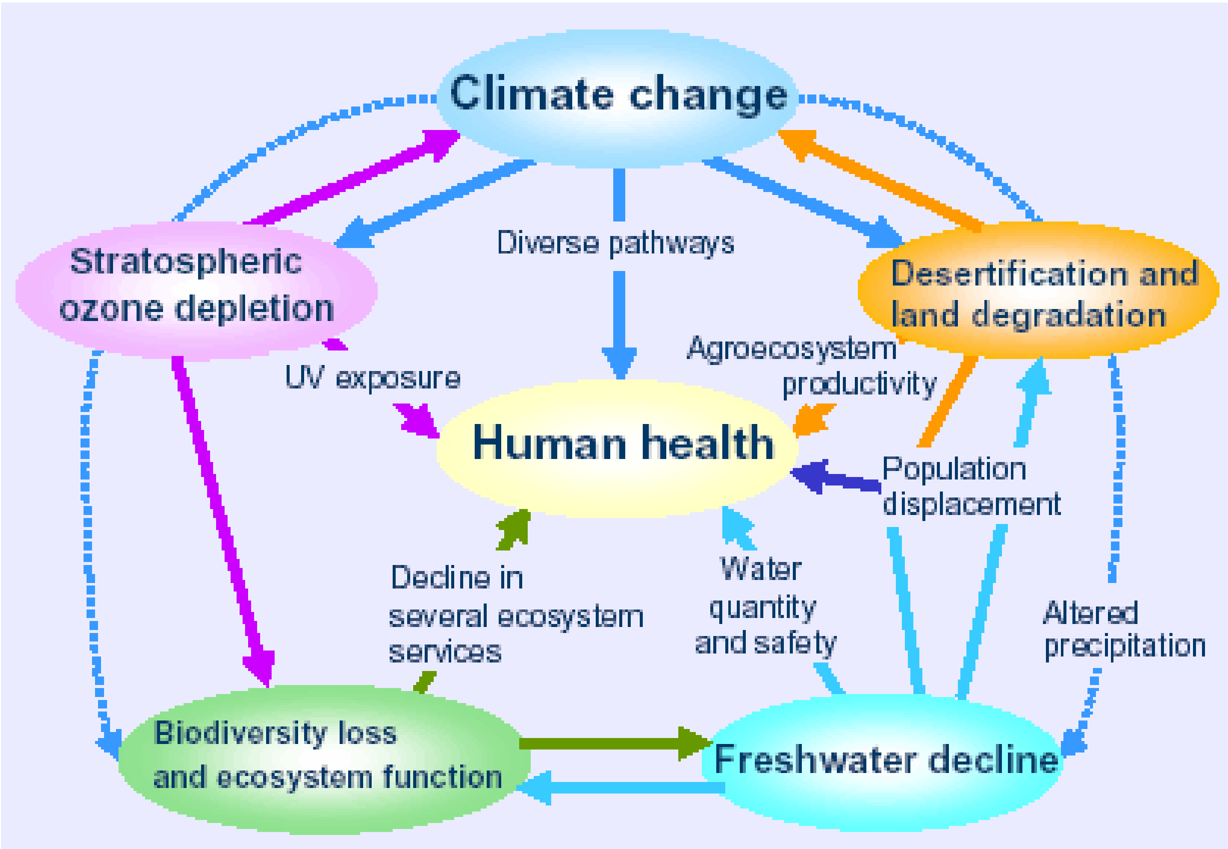
Positive Check - Recent Evidence on rise in mortality

Pollution

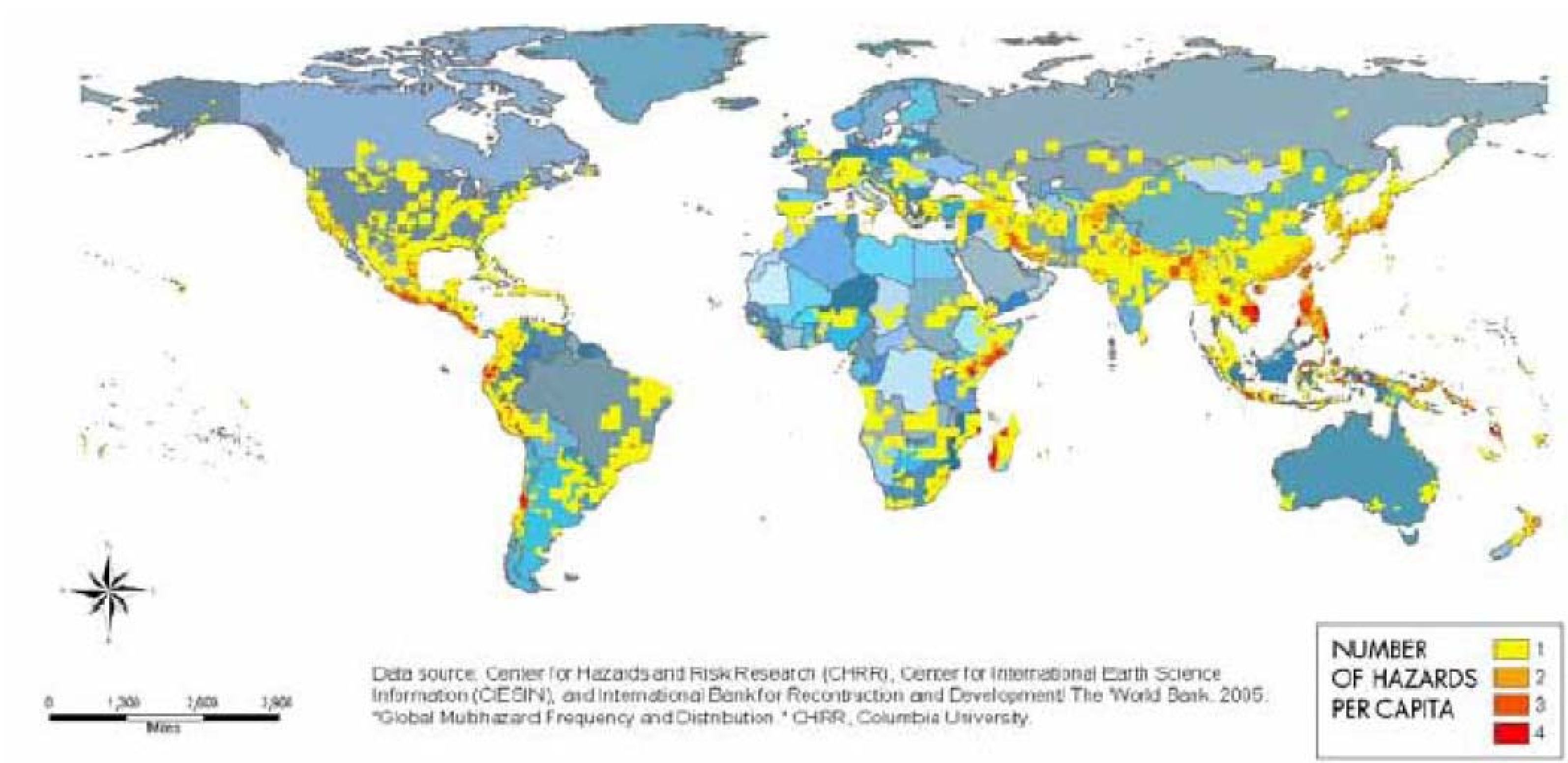
- **Air** Pollution (Samet et. Al 2000, Brunekreef and Holgate 2002)
- In Europe 350 000 premature deaths annually (CAFE and WHO 2004)

Climate change

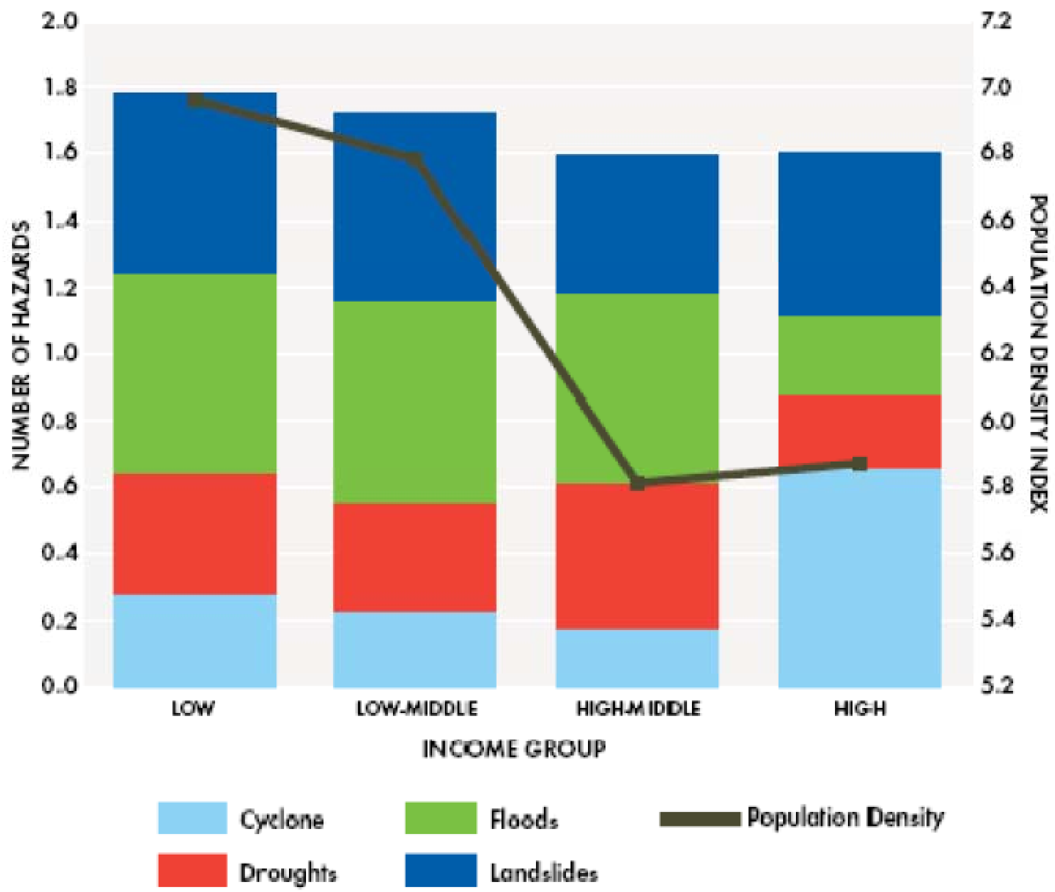
- Increase in malaria-areas (Tansier 2003)
- Weather extremes, lack of good trinking water, drough, population displacements (IPCC)
- Increase of risk for the great killers (malaria 83%, diarrhoea 17%, malnutrition 32%) (WHO 2003)
- 140 000 excess deaths annually (WHO 2010)


Climate change
Stratospheric ozone depletion
Diverse pathways
Desertification and land degradation
Agroecosystem productivity
UV exposure
Human health
Population displacement
Decline in several ecosystem services
Water quantity and safety
Altered precipitation
Biodiversity loss and ecosystem function
Freshwater decline

Mostly in South



Some types in North as well



Western Australia

The scenario for WA

- Expected average temperature increases of 0.5⁰C to 2⁰C.
- Increases in the number of days over 35⁰C in:
  South West of +1 to + 20 days (now 27 in Perth).
  North West of +10 to +90 days (now 54 in Broome and 156 in Halls Creek).
- Rainfall changes in:
  South-West of 2 to 20% reduction in annual rainfall with a 17% reduction in winter rain days and catchment runoff decreases of 5 to 40%.
  North-West of annual rainfall decreases of 1.5 to 3.5%.
- Sea-Level increases of 3 to 17 cm by 2030 and 25 to 75 cm by 2100.
- For Extreme Weather Events the following are generally accepted:
  - heatwaves - more per year
  - droughts - more frequent and severe
  - bushfires - increased risk
  - flooding - increased intensity
  - storms - increased intensity
  - tropical cyclones - increased intensity.

Prototype model

- Two channels from pollution to mortality
  - Through emissions E
  - Trough pollution stock S
- Acute versus cronic exposure
- Cronic exposure assumed here
- This is a long-run growth model

Prototype

Population growth

- Population growth *n=f-m*
- Constant fertility *f*
- Endogenous mortality *m*
- Critical pollution stock beound which populatin starts to decrease
- Population at time t

Responds to pollution stock S

$$n = n(S),\ n(0) > 0$$

$$n'(S) < 0,\ n\left(\hat{S}\right) = 0$$

$$L(t) = \exp \int_0^t n[S(\tau)]\, d\tau$$

Prototype

Pollution Stock S

Dictated by emissions E and abatement function *δ(S)*

$$\dot{S} = E - \delta(S)$$

- Inverted U abatement function

$$\delta(S)$$

- Carrying capacity larger than the critical level

$$\delta(0) = \delta\left(\tilde{S}\right) = 0$$

$$\delta'(0) > 0,\ \delta''(S) < 0,\ \delta'\left(\tilde{S}\right) < 0$$

- Allowing negative population growth

$$\tilde{S} > \hat{S}$$

Household optimization

- Utility from per capita emissions *E/L*

$$U = \int_0^\infty u\left[E(t)/L(t)\right]\; L(t)\; e^{-\rho t}\, dt$$

- Population at time *t*

$$L(t) = e^{\int_0^t n[S(\tau)]d\tau}$$

- The integrand

$$u\left[E(t)/L(t)\right] e^{-\int_0^t \{\rho - n[S(\tau)]\}d\tau}$$

- Equation of motion

$$\dot{S} = E - \delta(S)$$

The Uzawa-trick: solution in virtual time

Constant discount factor

$$\Delta(t) = \int_0^t \{\rho - n[S(\tau)]\} d\tau$$

$$\frac{d\Delta(t)}{dt} = \rho - n[S(t)]$$

$$dt = \frac{d\Delta(t)}{\rho - n[S(t)]}$$

Transformation of the model

$$U = \int_0^\infty u[E(t)/L(t)]\, e^{-\int_0^t \{\rho - n[S(\tau)]\} d\tau}\, dt$$

$$\dot{S} = E - \delta(S)$$

$$U = \int_0^\infty \frac{u(E/L)}{\rho - n(S)} \cdot e^{-\Delta} \cdot d\Delta$$

$$\mathring{S} = \frac{dS}{d\Delta} = \frac{dS}{dt}\frac{dt}{d\Delta} = \frac{E - \delta(S)}{\rho - n(S)}$$

The Uzawa-trick: solution in virtual time

Solution in virtual time

$$H(S,E,\lambda) = \frac{1}{\rho - n(S)} \{u(E/L) + \lambda(\Delta)[E - \delta(S)]\}$$

$$\frac{\partial H(S,E,\lambda)}{\partial E} = 0 \Longleftrightarrow -u'(E/L) = \lambda(\Delta) \cdot L,$$

$$\mathring{\lambda} = \frac{d\lambda(\Delta)}{d\Delta} = -\frac{\partial H}{\partial S} + \lambda(\Delta)$$

$$\lim_{\Delta \to \infty} \lambda(\Delta)\; e^{-\Delta}\; S = 0$$

Transformed to natural time

CIES utility functionN

$$u(E/L) = \left[(E/L)^{1-\theta} - 1\right] / (1-\theta), \; \theta \neq 1$$

$$\frac{\dot{E}}{E} = \frac{1}{\theta} \left\{ \frac{n'}{\rho - n} \left[\frac{\theta E}{\theta - 1} - \delta \right] - (\delta' + \rho - \theta n) \right\}$$

$$\dot{S} = E - \delta(S)$$

Phase lines:

$$\frac{\dot{E}}{E} = 0 \Leftrightarrow E = \frac{\theta - 1}{\theta} \left\{ \delta + \frac{\rho - n}{n'} (\delta' + \rho - \theta n) \right\}$$

$$\dot{S} = 0 \Leftrightarrow E = \delta(S)$$

Sustainability in a steady state

- Per capita consumption and population are substitutes
- Population growth can be of any sign
- Emissions *E* constant
- Per capita emissions *E/L* grow at rate
- Golden Rule emissions are exceeded (dynamic efficiency)

$$U = \int_0^\infty u\left[E\left(t\right)/L\left(t\right)\right]\ L\left(t\right)\ e^{-\rho t}\ dt$$

$$n^* = n(S^*)$$

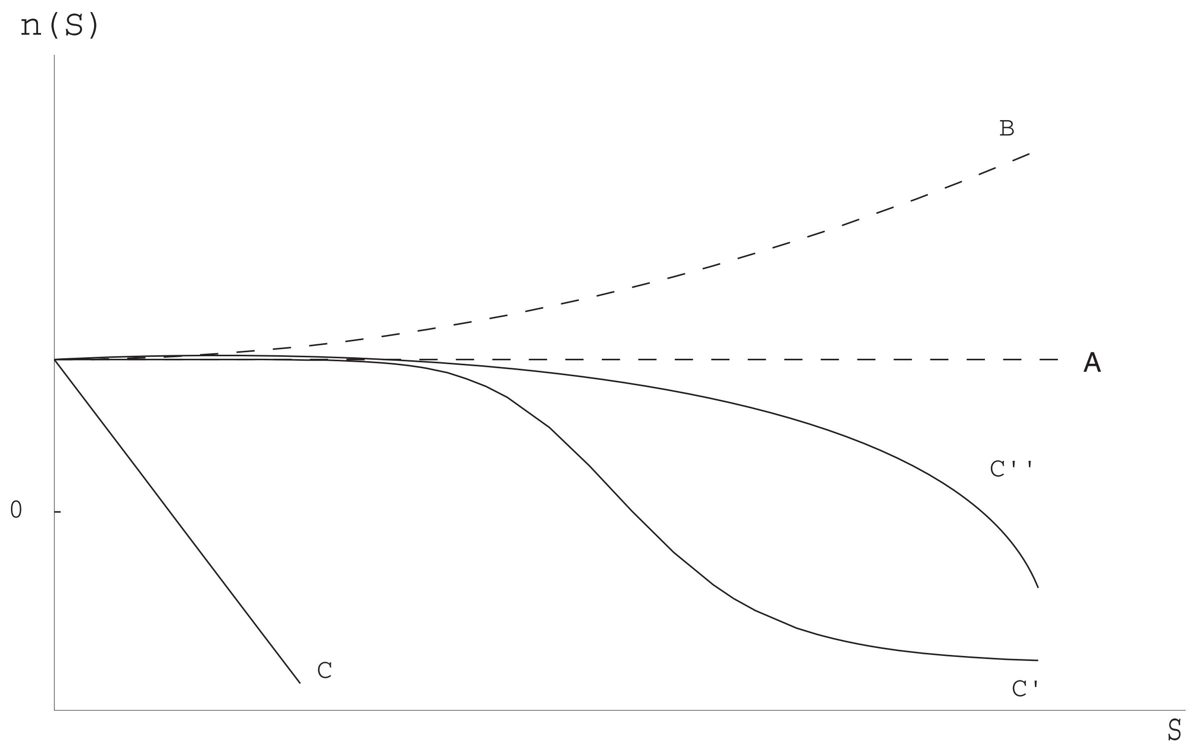
$$-n^* = n(S^*)$$

if $n^* < 0$, then

$$\lim_{t-\infty} L(t) = 0$$

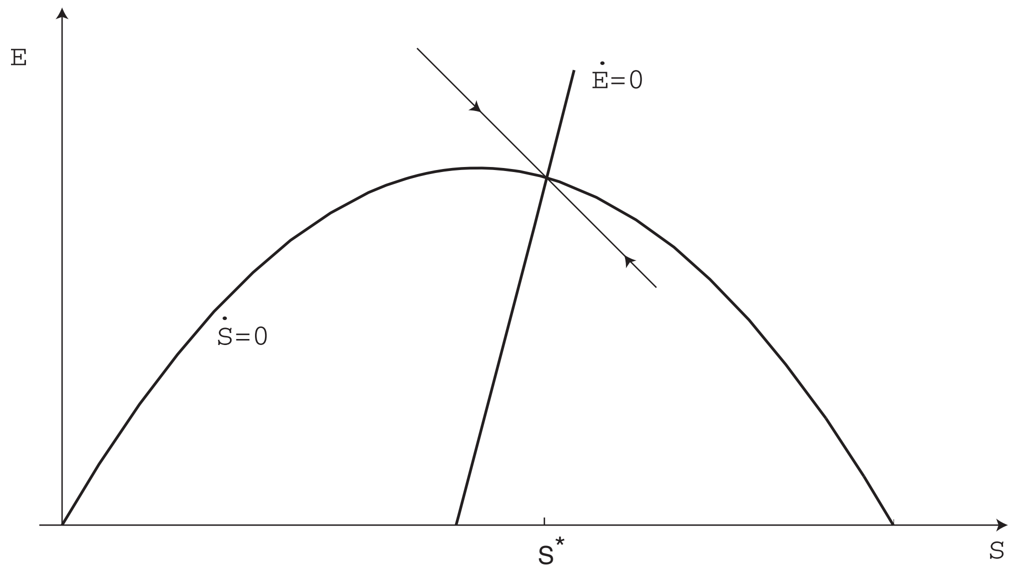
How mortality responds to pollution?
$n(S) = f - m(S)$

- Report of Rome 1973:



Linear C: single steady state

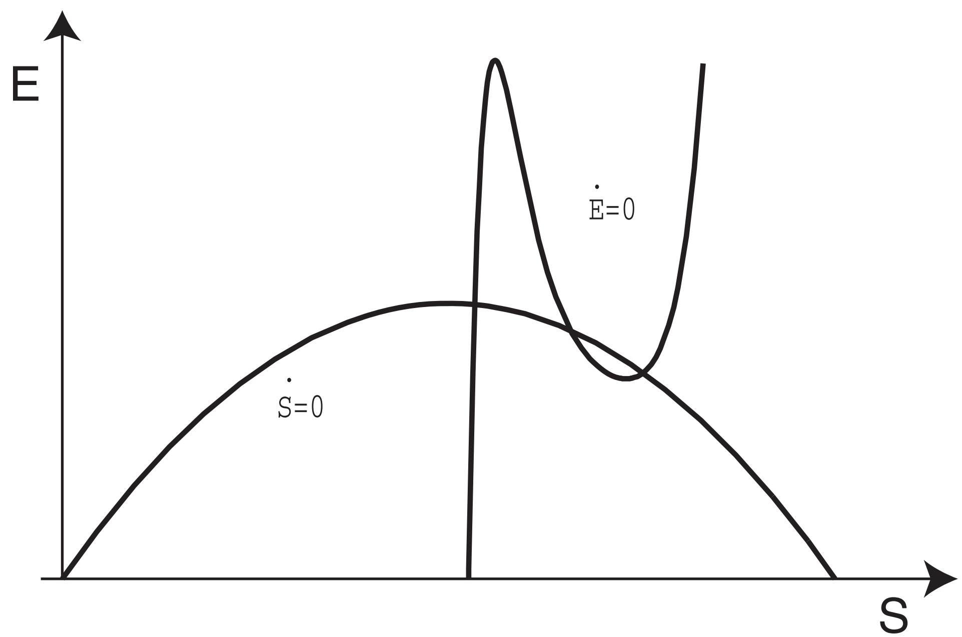
$$n(S) = \beta - \eta S, \quad \beta,\ \eta > 0$$



Non-linear C’: mortality threshold.

$$\frac{\dot{E}}{E} = 0 \Leftrightarrow E = \frac{\theta - 1}{\theta}\left\{\delta + \frac{\rho - n}{n'}(\delta' + \rho - \theta n)\right\}$$

- The phase line curved
- Multiple steady states possible



Consumption augmenting exogenous technical progress

$$C = e^{xt} E$$

$$C/L = e^{xt} E \ / \ L$$

$$\frac{\dot{E}}{E} = 0 \Leftrightarrow E = \frac{\theta - 1}{\theta} \left\{ \delta + \frac{\rho - n}{n'} \left[\delta' + (\theta - 1)\, x + (\rho - \theta n) \right] \right\}$$

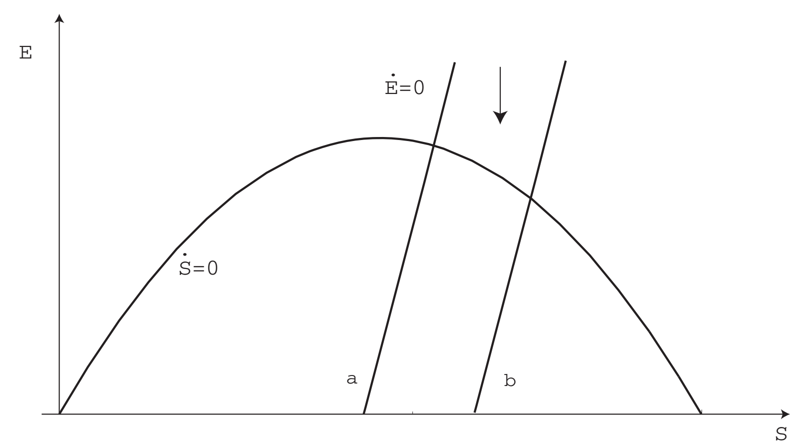
$$\dot{S} = 0 \Leftrightarrow E = \delta(S)$$

$$\frac{\partial E}{\partial x} \, |_{\dot{E}/E=0} = \frac{(\theta - 1)^2 \, (\rho - n)}{\theta n'} < 0$$

$$\frac{\partial E}{\partial x}\Big|_{\dot{E}/E=0} = \frac{(\theta-1)^2(\rho-n)}{\theta n'} < 0 \qquad U = \int_0^\infty u\left[E(t)/L(t)\right]\; L(t)\; e^{-\rho t}\, dt$$

- E-phaseline shift down
- S* increases
- Population growth decreases
- Population growth may get negative

- Linear population function:



Calibrated examples

Logistic abatement function

$$\delta(S) = r\,S(1 - \frac{S}{\tilde{S}})$$

Parameter	Value
$\tilde{S}$	1000
r	0.2
θ	3
ρ	0.04
β	0.01
η	0.000015
$\hat{S}$	667
x	0.00 and 0.02

Linear population function

$$n(S) = \beta - \eta\,S$$

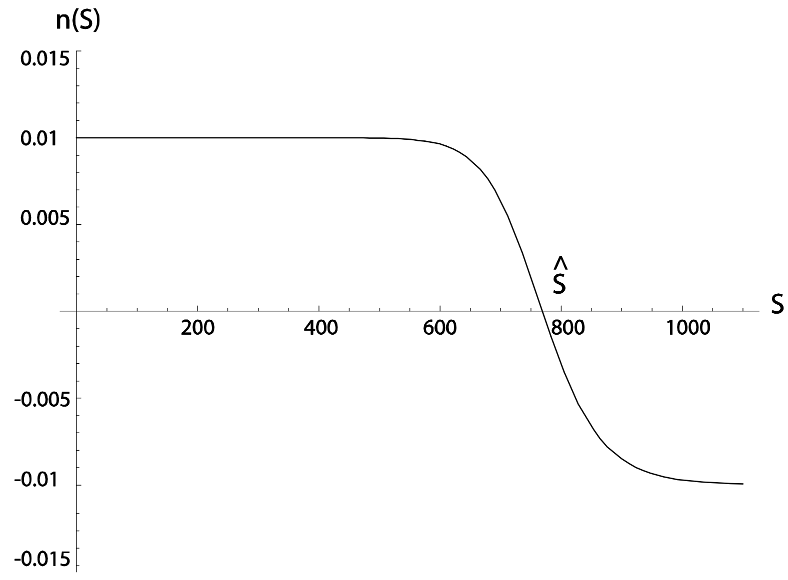
Results	x = 0.00	x = 0.02
S^*	618	727
$n(S*)$	0.007%	–0.08%
Doubling time	930	–794

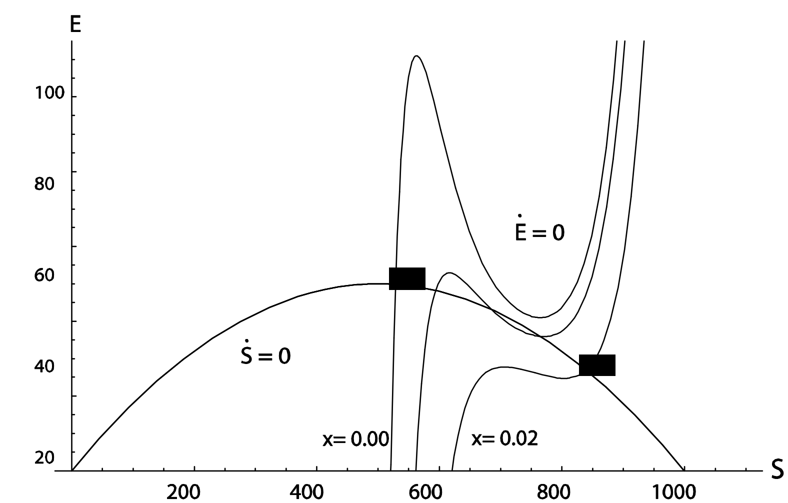
Threshold population function

$$n(S) = \beta - \frac{\alpha}{1 + (\mu\, S)^{-\gamma}}$$

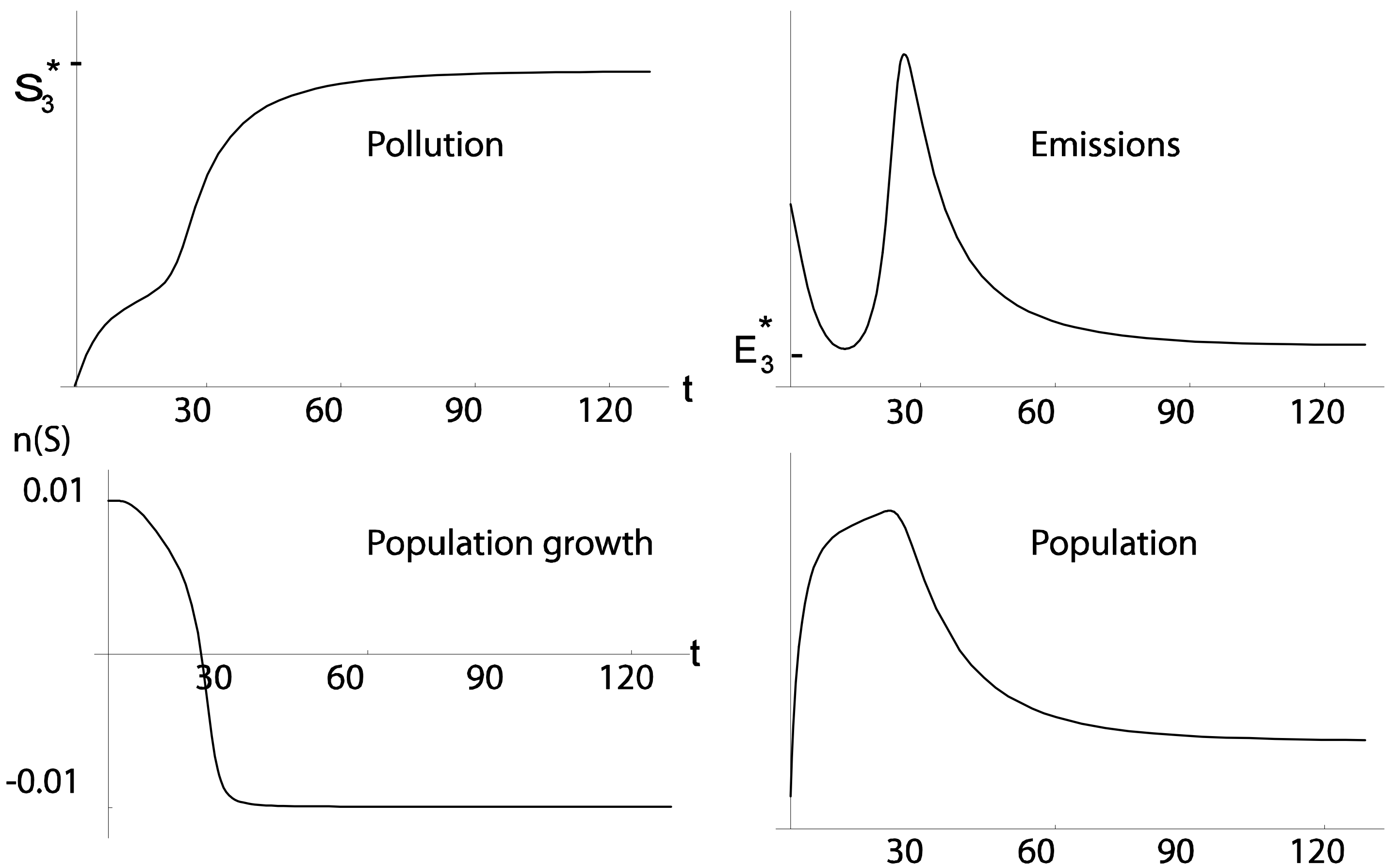
Parameter	Value
$\tilde{S}$	1000
r	0.2
θ	3
ρ	0.04
β	0.01
η	0.000015
$\hat{S}$	667
x	0.00 and 0.02

Results	x = 0.00	x = 0.02
S^*	528	805
$n(S*)$	0.99%	−0.3%
Doubling time	70	−231





Time paths


S_3^*
Pollution
t
30
60
90
120
Emissions
E_3^*
t
30
60
90
120
n(S)
0.01
Population growth
t
30
60
90
120
-0.01
Population
t
30
60
90
120

Discussion

- Environmental mortality in a prototype of pollution.
- Environment public good.
- Incentives for births private?
- Do people care?
- Birth policies difficult after some level of fertility: This has been reached in most countries
- Child-number is a private area