An Economic Model of Oil Exploration and Extraction

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III. A Model

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I. Introduction

- Greiner/Semmler Book: „Global Environment, Economic Growth and Natural Resources,“ OUP, 2008
- **Renewable Resource** (Clark): Growth rate of the resource in interaction with other resources (Lotka-Volterra system), see paper with L.Bernard and A. Greiner for agricultural products
- **Non-Renewable Resources** (Hotelling): stock of exhaustible resource is given, extraction rate monotonically decreasing, competitive pricing, and prices increase montonically with interest rate
- We here treat **oil (fossil fuel)** as representing something between a non-renewable and renewable resource
- We present results from **infinite** and **finite horizon**
- We **sketch an optimal growth model** with fossil fuel and renewable energy (and how **transition** to renewable energy can be accelerated in a market economy by tax and subsidies)

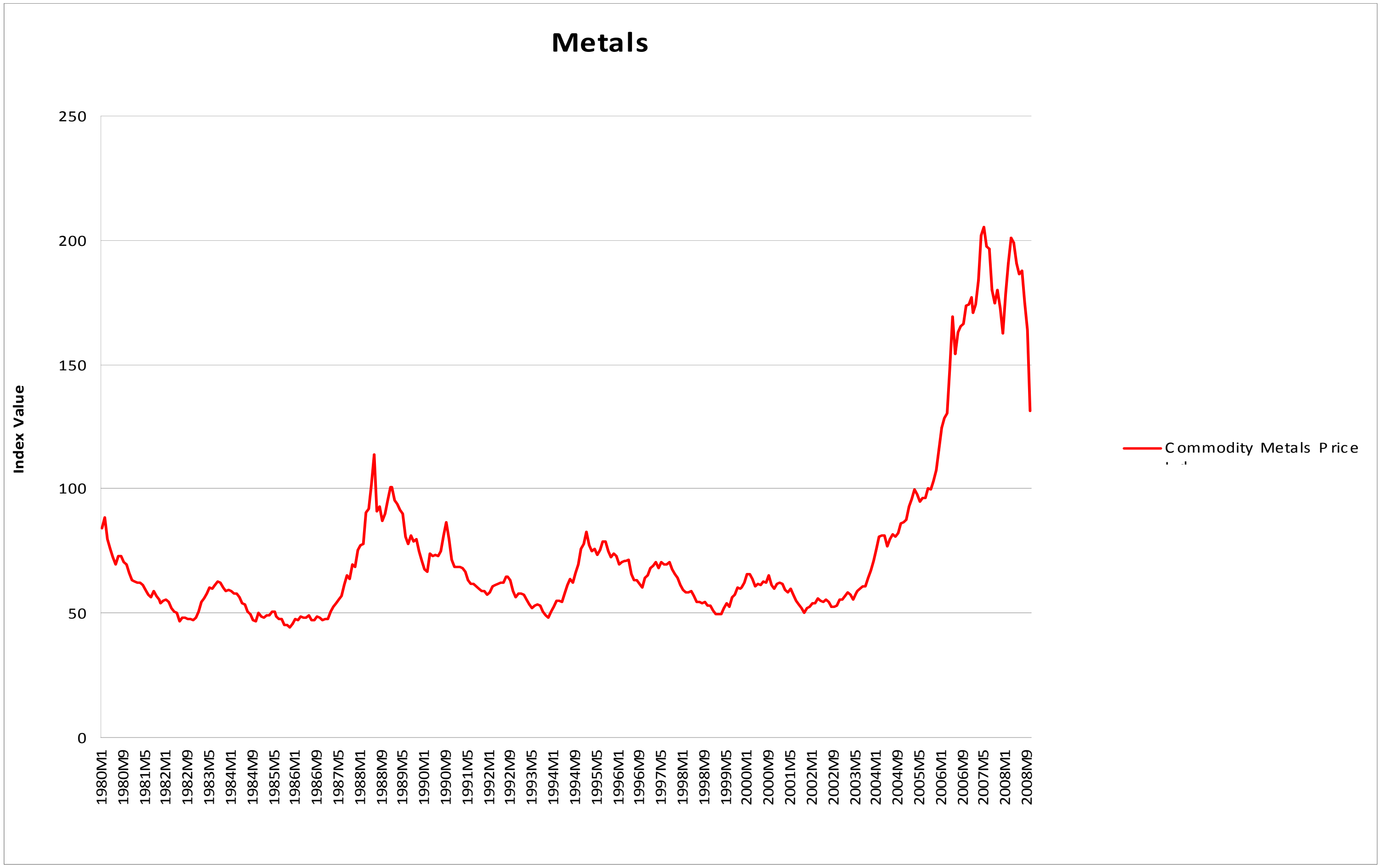
II. Empiral Facts: Price Volatility and Trend

⇒ **Reneweable** commodities show high volatility and no trend

=> **Non-renewable** commodities (fossil fuel , metals) show higher volatility and trend

⇒ In particular **oil shows U-shaped price trend**

II. Empirical Facts: Volatility and Trend



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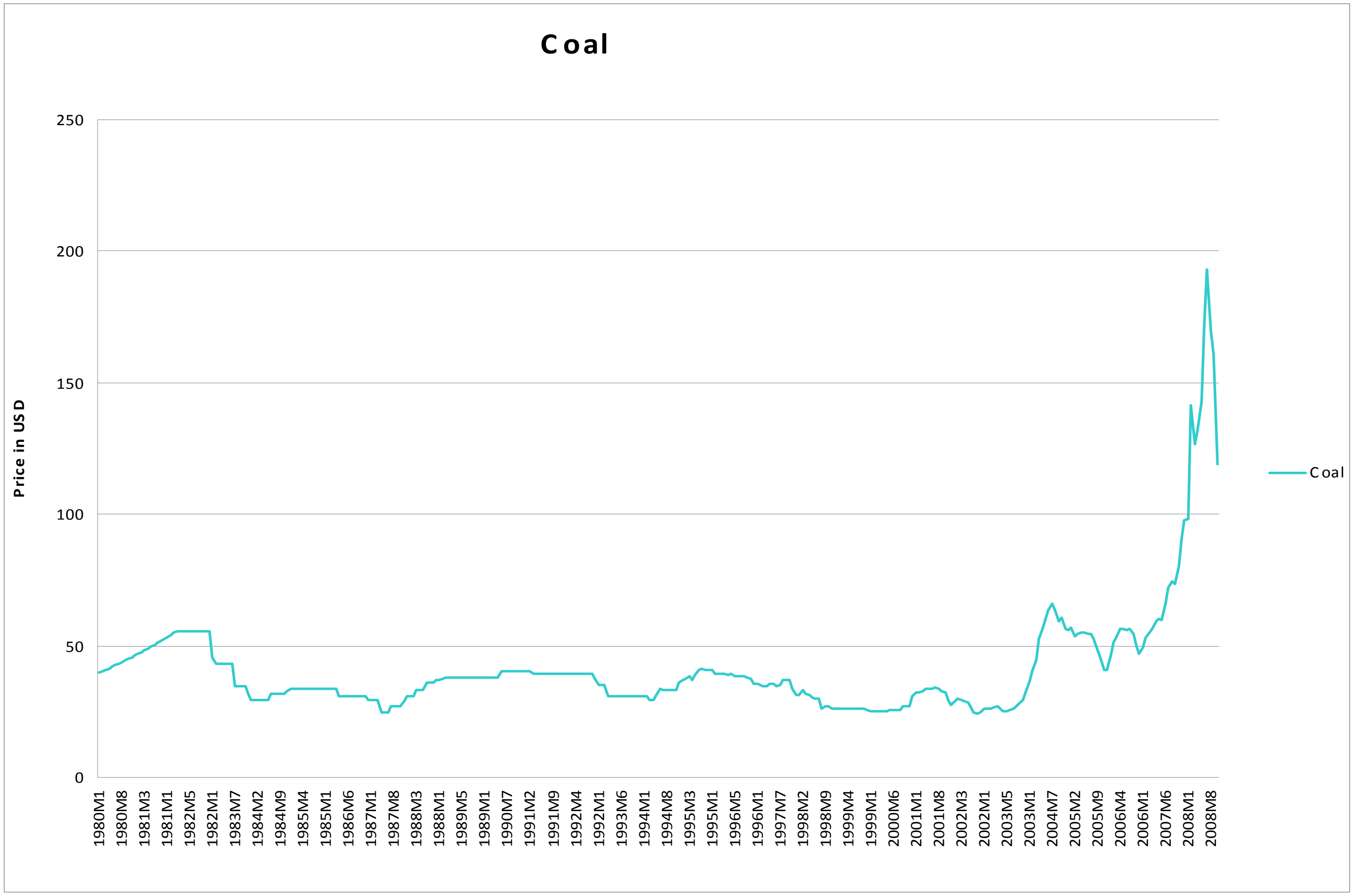
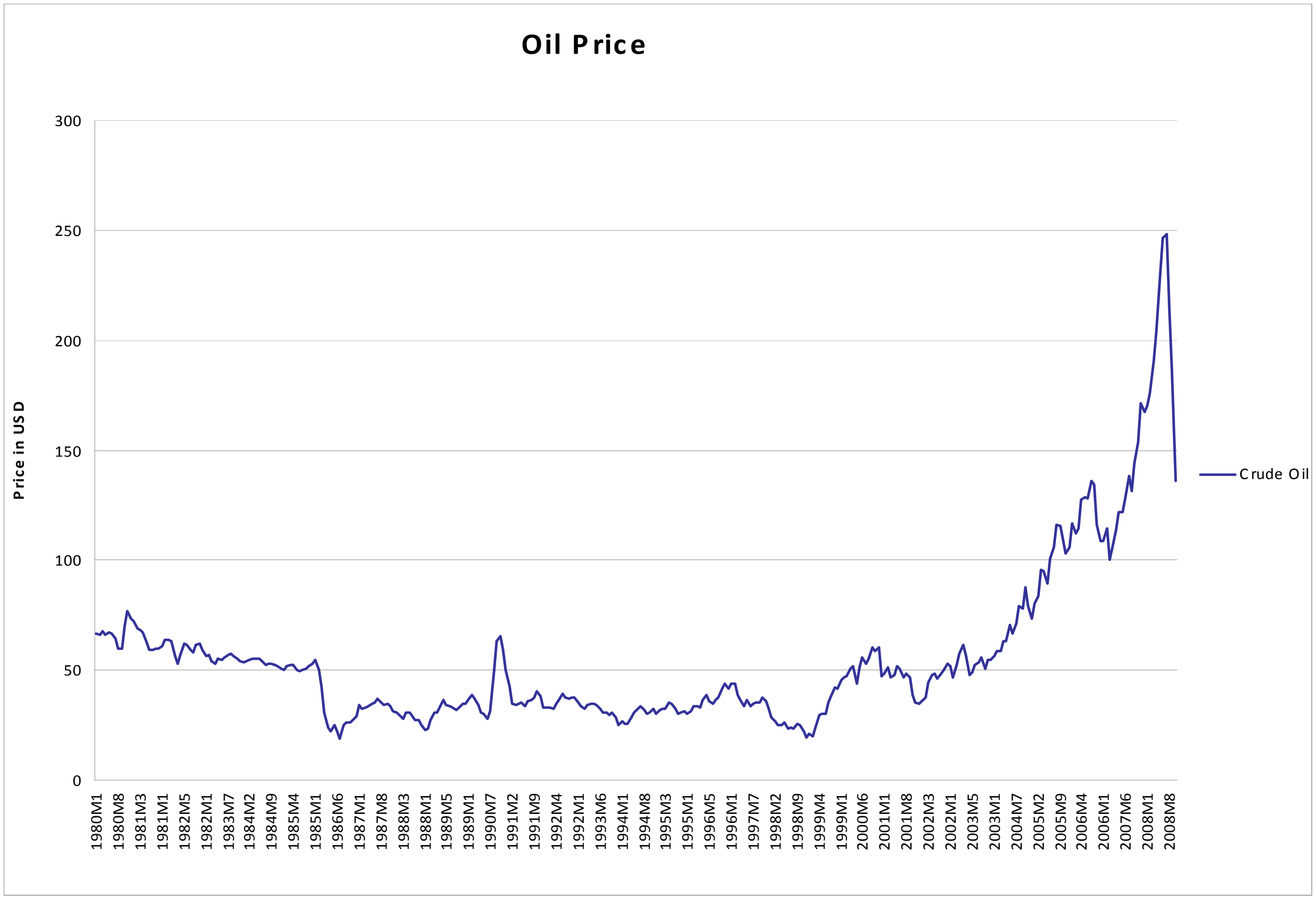
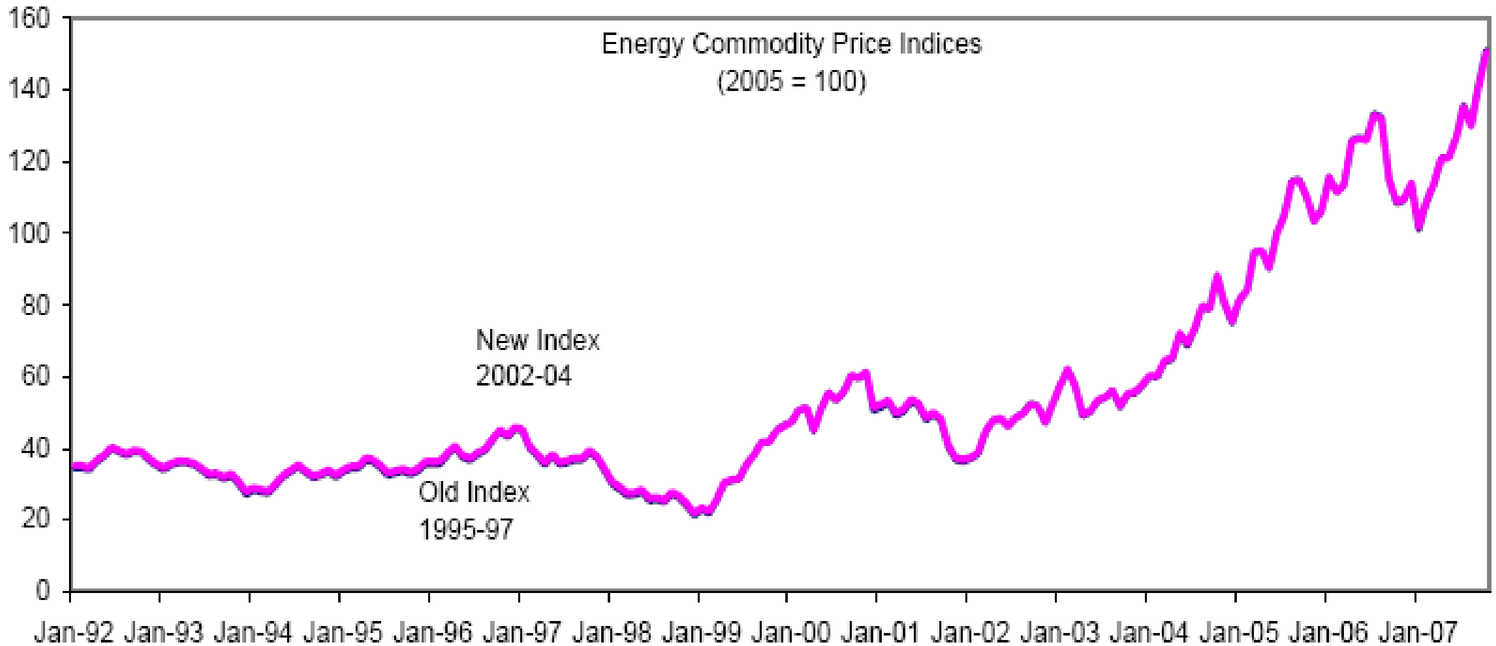


II. Empirical Facts: Volatility and Trends



II. Empirical Facts: Volatility and Trends

- IMF Energy Commodity Price Index



Source: IMF Energy and Commodities Surveillance Unit.

II. Empirical Facts: Historical Trends and Volatilities, 1980 - 2008

	Agricultural Commodities			Energy				Metals				Stocks	U.S. GDP
	Commodity Food Price Index[1]	**Commodity Agricultural Raw Materials Index[2]**	**Rice[3]**	**Crude Oil[4]**	**Crude Oil[11] (LT)**	**Coal[5]**	**Natural Gas[6]**	**Commodity Metals Price Index[7]**	**Copper[8]**	**Nickel[9]**	**Gold[10]**	**Dow Jones**	**Industrial Production Index**
Mean price change per month	0.03%	0.07%	0.13%	0.21%	0.30%	0.32%	0.50%	0.13%	0.18%	0.18%	0.03%	0.69%	0.28%
Volatility per month	2.72%	3.10%	6.26%	8.17%	9.50%	5.09%	6.35%	4.39%	6.32%	8.44%	4.93%	4.42%	0.66%
Mean price change per quarter	0.10%	0.20%	0.40%	0.62%	0.89%	0.96%	1.52%	0.39%	0.55%	0.53%	0.10%	2.07%	0.83%
Volatility per quarter	4.70%	5.38%	10.84%	14.15%	16.45%	8.82%	11.00%	7.61%	10.95%	14.62%	8.53%	7.66%	1.14%

All samples from monthly data, 1980-2008. Source IMF.

Exception: The time series"Oil" are nominal annual oil prices from 1860 -2007.

[1]includes Cereal, Vegetable Oils, Meat, Seafood, Sugar, Bananas, and Oranges Price Indices

[2] includes Timber, Cotton, Wool, Rubber, and Hides Price Indices

[3] Five percent broken milled white rice, US$ per metric tonne

[4]simple average of three spot prices; Dated Brent, West Texas Intermediate, and the Dubai Fateh

[5] Australian thermal coal, 12000- btu/pound, less than 1% sulfur, 14% ash, FOB Newcastle/Port Kembla, US$ per metric tonne

[6] Russian Natural Gas border price in Germany, US$ per thousands of cubic meters of gas

[7]includes Copper, Aluminum, Iron Ore, Tin, Nickel, Zinc, Lead, and Uranium Price Indices

[8]grade A cathode, LME spot price, CIF European ports, US$ per metric tonne

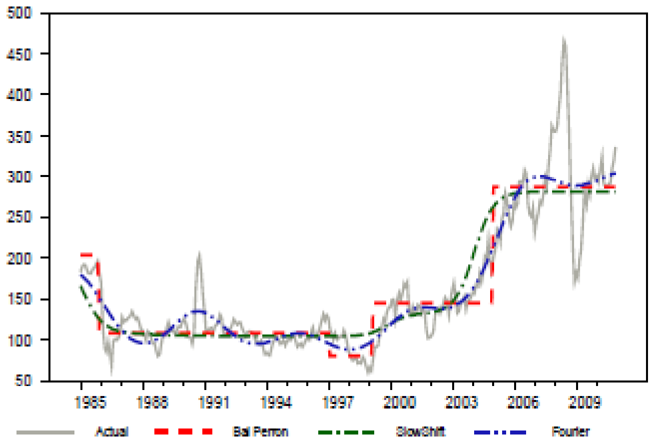
[9]melting grade, LME spot price, CIF European ports, US$ per metric tonne

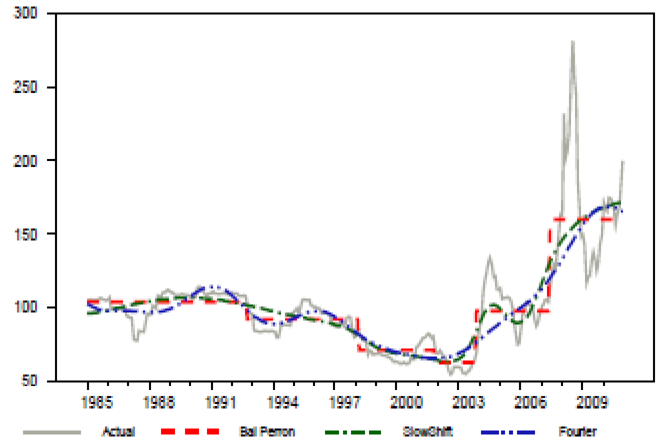
[10]Price in US Dollars US$/oz

[11]Nominal annual oil prices, longterm: 1860 - 2007

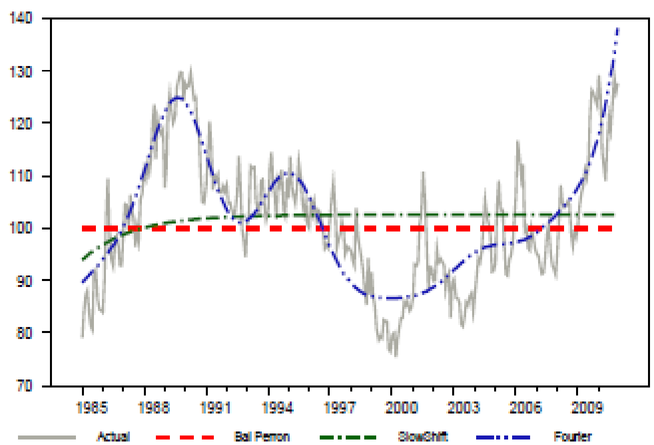
II. Empirical Facts: Historical Trends and Volatilities, 1980 – 2010
Econometric work: Enders and Holt (2011), shifting mean

Non-Renewable:Oil (left), Coal (right)





Renewable:Food



III. A Model: Conjectures on Price Trends

- **Renewable Resources**: Price volatility, but trend is uncertain
- **Non-Renewable** (…Oil); 1) Hotelling (1931): price monotonically rises, 2) Recent views: price first falls, then rises, Pindyck (1978)
- Note: We do not study volatility, see Pindyck (2001, 2003, 2004)

III. A Model: Hotelling on Non-Renewable Resources

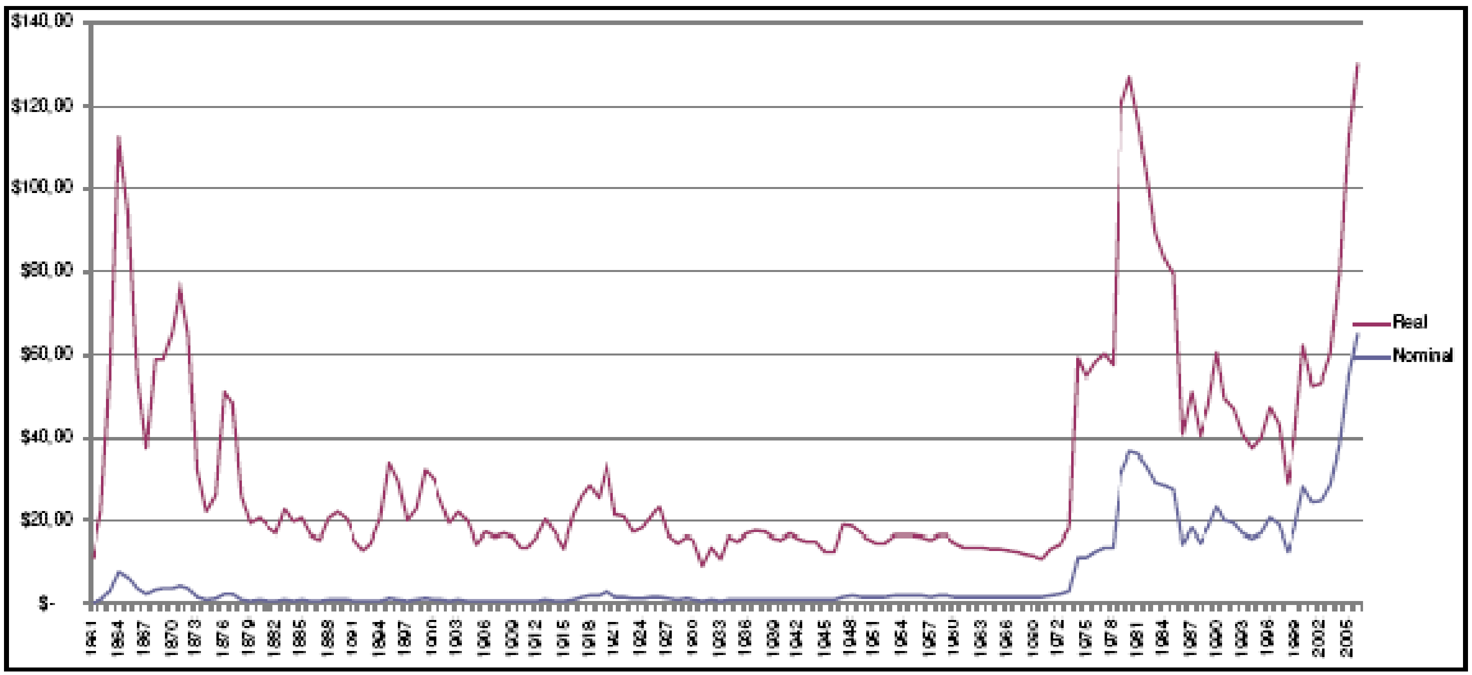
- **Non-Renewable Resource (1. Hotelling, 1931: Price rises with interest rate, 2. Pindyck, 1978: Price may first deline, before rising)**

Running down the stock ($x_0 > 0$)

$$
\begin{aligned}
\dot{x} &= -u(x) \\
& \quad x_0 - y \\
y &= \int_0^\infty u dt
\end{aligned}
$$

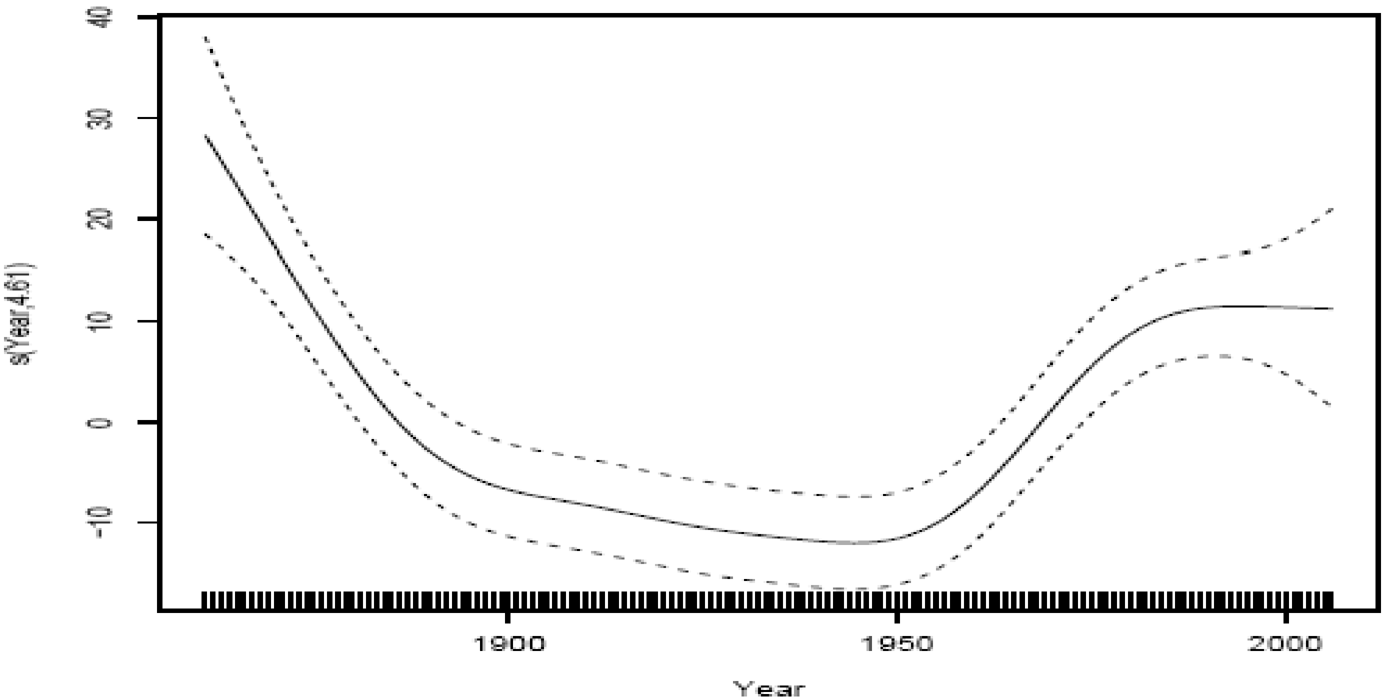
III. A Model: Conjecture -- First Falling and then Rising Oil Price

- Nominal and real oil price, since 1880



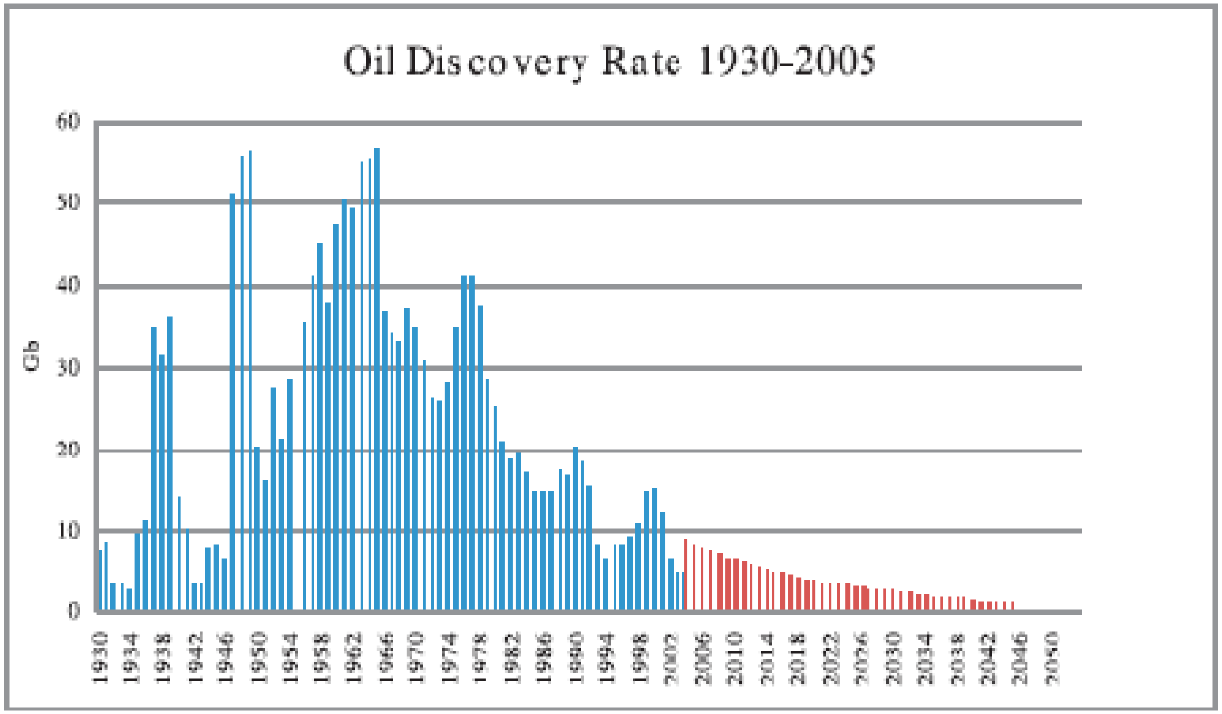
IV. A Model: Conjecture -- First Falling and then Rising Oil Price

- Non-parameteric estimation of the oil price (Penalized Splines)



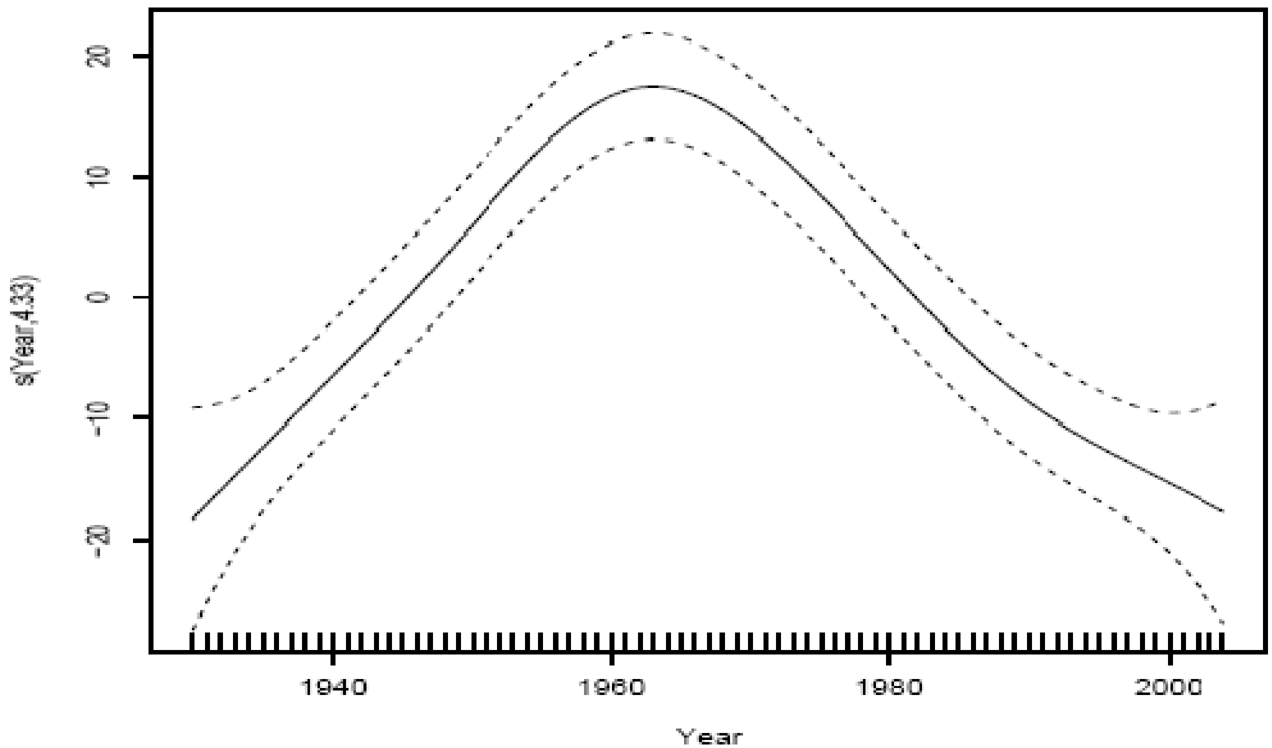
IV. A Model: Conjecture--First Falling and then Rising Oil Price

- Discovery rates



IV. A Model: First Falling and then Rising Oil Price

- Non-parameteric estimation of discovery rates (Penalized Splines)



IV.1. A Model: Oil Discovery and Extraction—Infinite Horizon

Oil discovery and extraction model with discovery rates (using DP, Grüne/Semmler, 2004)

$$\max_u \int_0^\infty e^{-rt} \left(p(u,y) - C(x_o - y)\right) u \, dt,$$

$$\begin{aligned} \dot{x}^k &= -u + f\left(x_o - y - x^k\right), \qquad x^k(0) > 0 \\ \dot{y} &= u, \qquad y(0) \geq 0 \\ \lim_{t\to\infty} x^k &\geq 0 \end{aligned}$$

IV.1. A Model: Oil Discovery and Extraction– Infinite Horizon

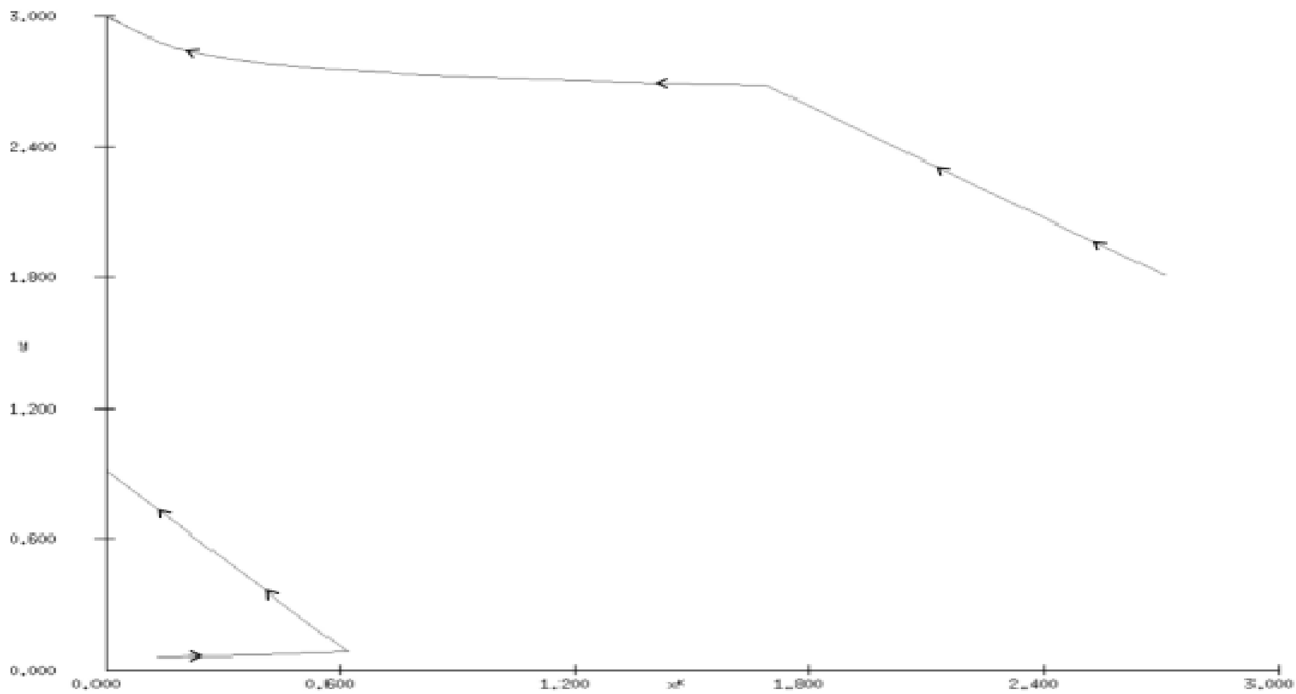
- Cost (exploration and extraction costs) and price

$$C(x_0 - y) = (\phi/2)(x_0 - y)^{-2},\ \phi > 0$$

$$p(u, y) = \left(\frac{1}{\gamma + \eta u - \mu y}\right)^{\alpha},\ \alpha > 0,\ \gamma > 0,\ \eta > 0,\ \mu \geq 0$$

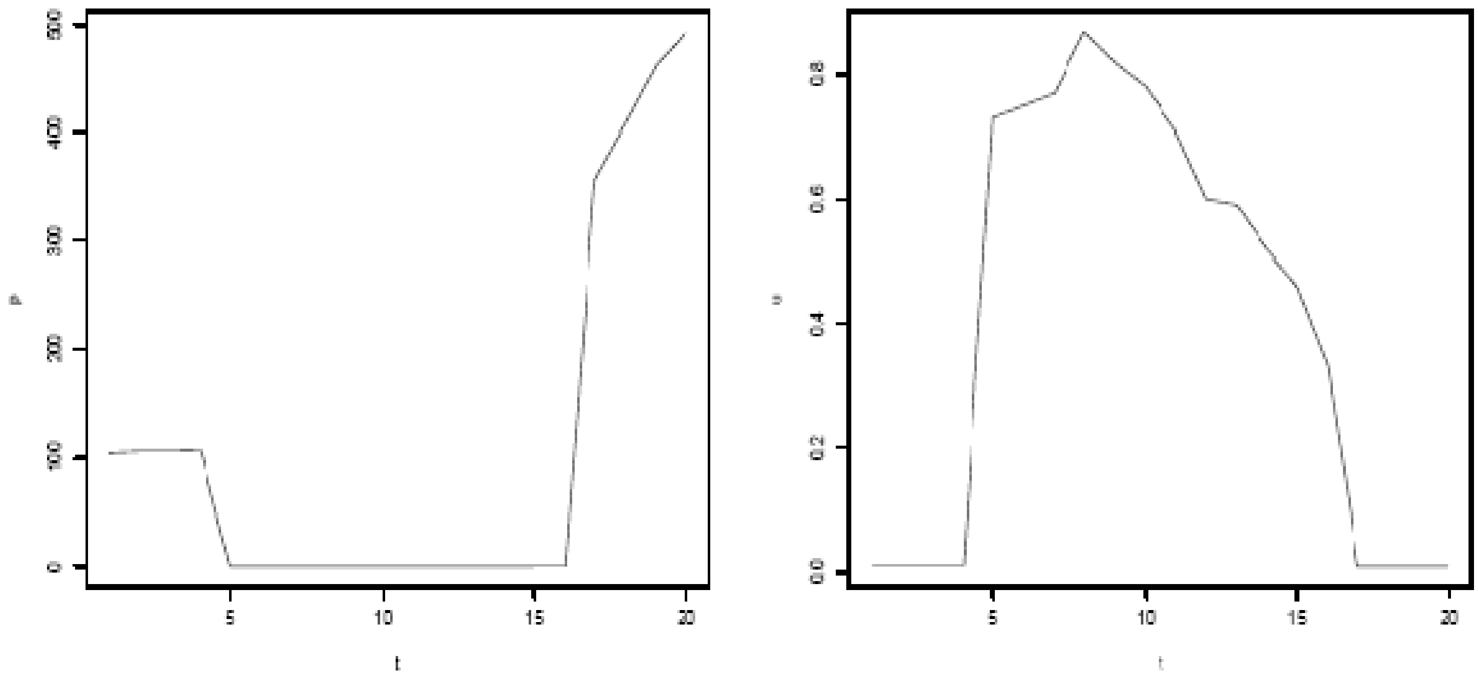
IV.1. A Model: Oil Discovery and Extraction– Infinite Horizon

- Small and large known reserves



IV.1. A Model: Oil Discovery and Extraction– Infinite Horizon

- Oil Price (left) and extraction rate (right) for small known reserves (Sinn`s green paradox? see left)



IV.2. A Model: Oil Discovery and Extraction– Finite Horizon (Maurer et al., 2010)

The model with finite horizon (using Büskens and Maurer, 2000)

T

$$\max_{u} \int_0 e^{-rt} \left(p(u,y) - C(x_o - y)\right) u \, dt,$$

$$\begin{aligned} \dot{x}^k &= -u + f\left(x_o - y - x^k\right), \qquad x^k(0) > 0 \\ \dot{y} &= u, \qquad y(0) \geq 0 \\ \lim_{t\to\infty} x^k &\geq 0 \end{aligned}$$

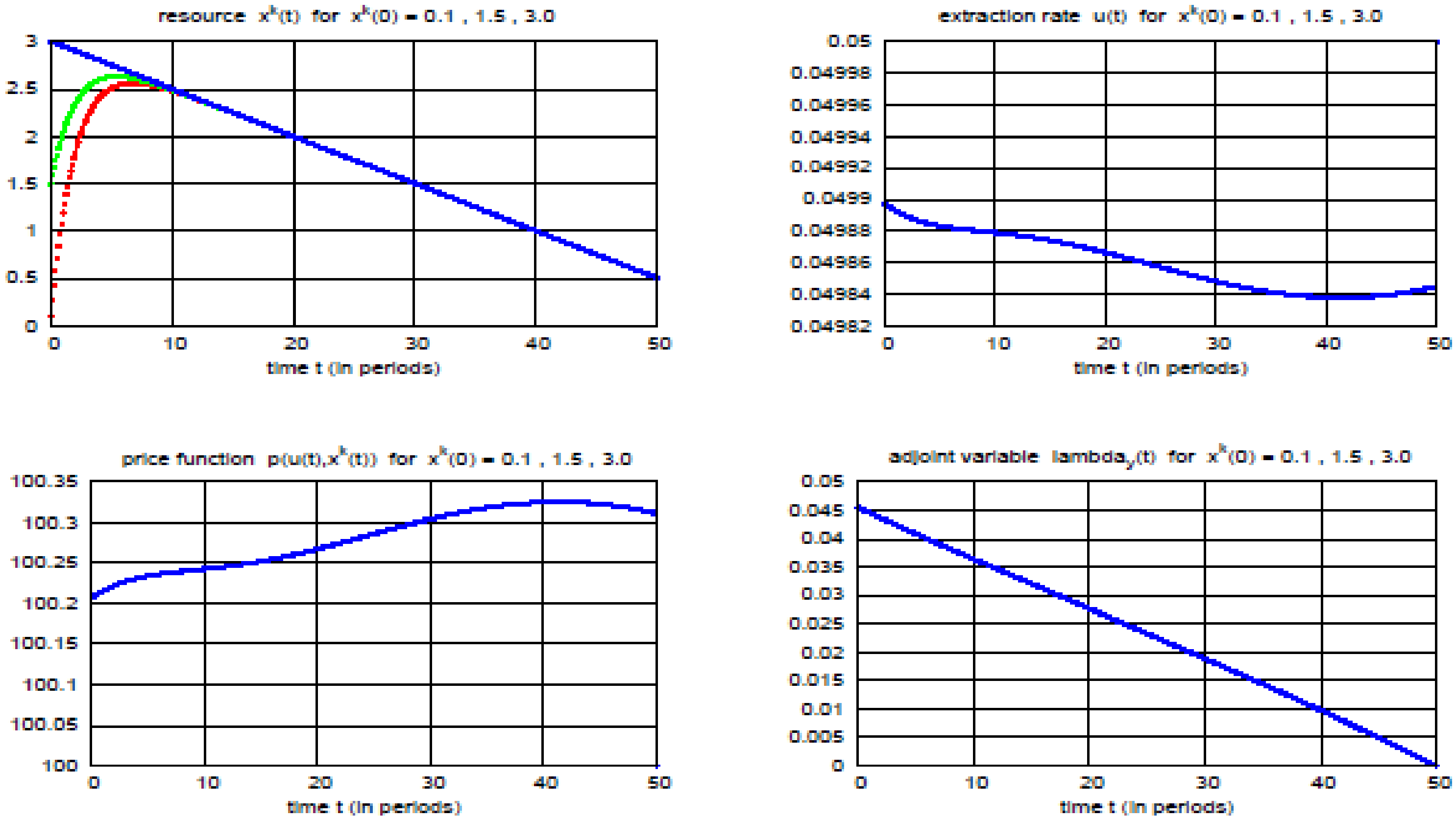
for $0 \leq t \leq T$

$$p(u, x^k, y) = \left(\frac{1}{\gamma + w_u \cdot u + w_x \cdot x^k - \mu y}\right)^{\alpha}$$

IV.2. A Model: Oil Discovery and Extraction–Finite Horizon

$$p(u, x^k, y) = \left(\frac{1}{\gamma + w_u \cdot u + w_x \cdot x^k - \mu y} \right)^{\alpha}$$

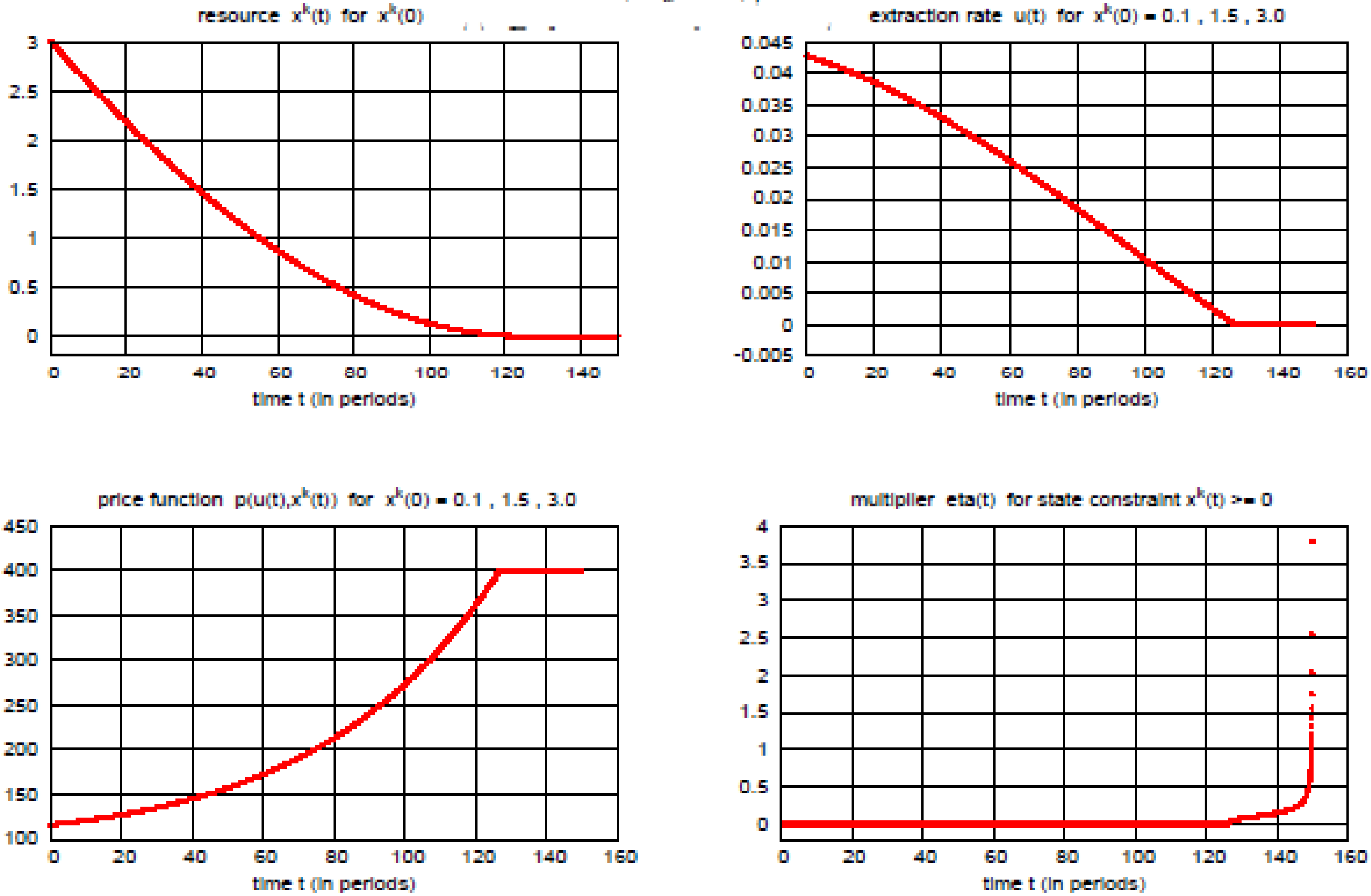
With finite horizion: Case $T = 50, w_x = 0, \mu = 0$ $\quad w_u = 1$



IV.2. A Model: Oil Discovery and Extraction–Finite Horizon

$$p(u, x^k, y) = \left(\frac{1}{\gamma + w_u \cdot u + w_x \cdot x^k - \mu y} \right)^{\alpha}$$

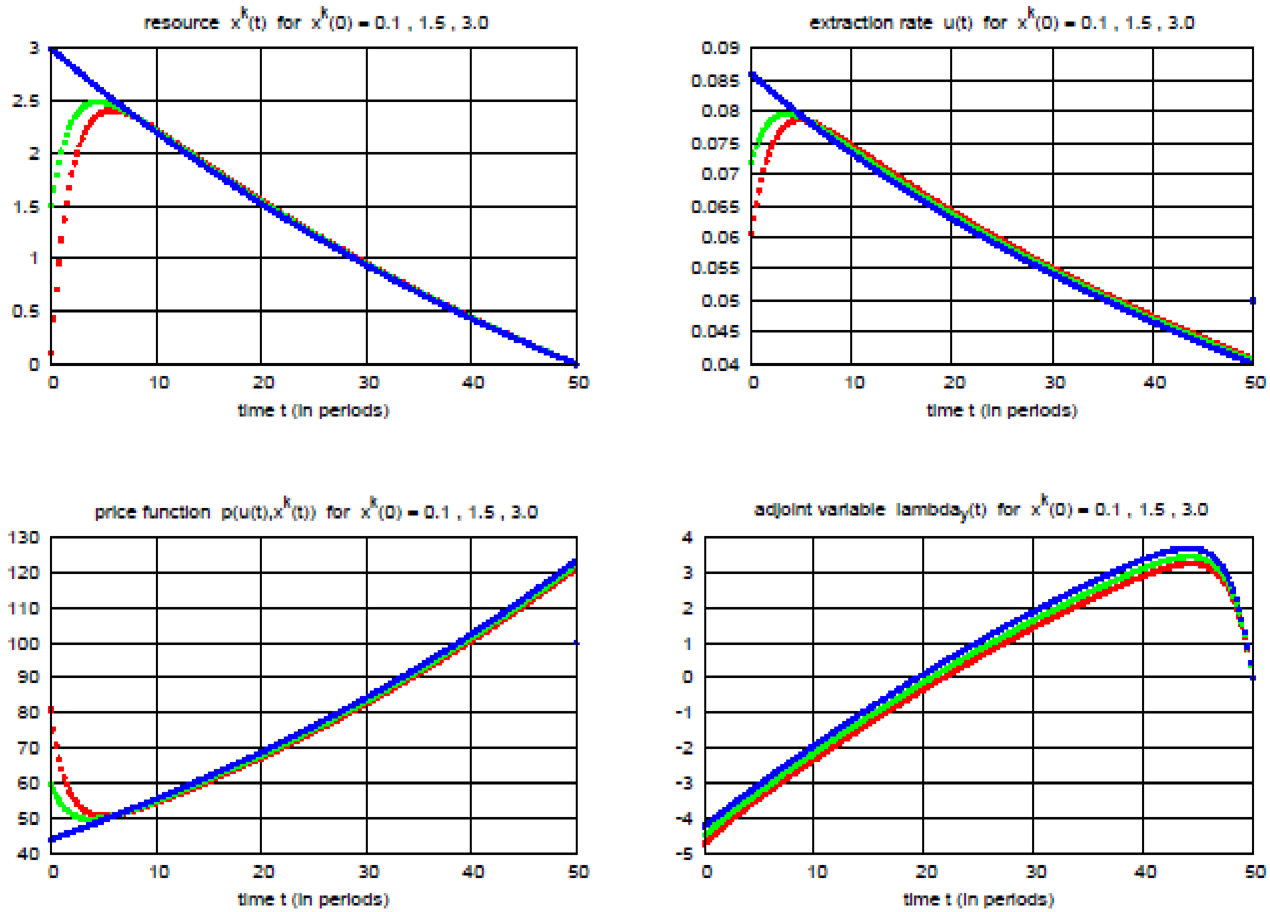
With finite horizion: Case $T = 150,\ w_x = 0,\ \mu = 0 \qquad w_u = 1$



IV.2. A Model: Oil Discovery and Extraction–Finite Horizon

$$p(u, x^k, y) = \left(\frac{1}{\gamma + w_u \cdot u + w_x \cdot x^k - \mu y} \right)^{\alpha}$$

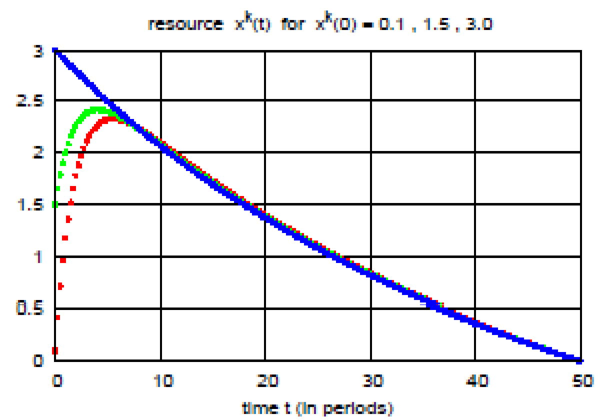
With finite horizion: Case $T = 50$, $w_x = 0.005$, $\mu = 0$, $w_u = 1$

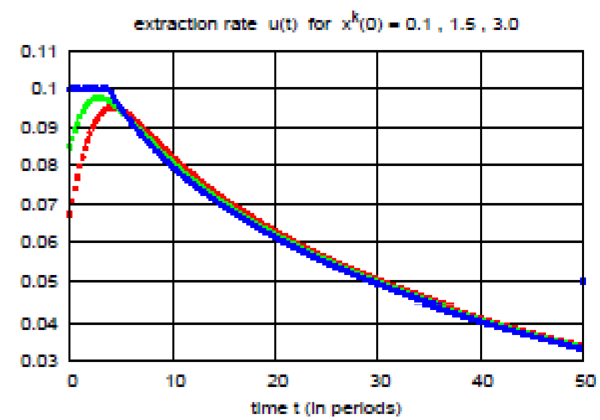


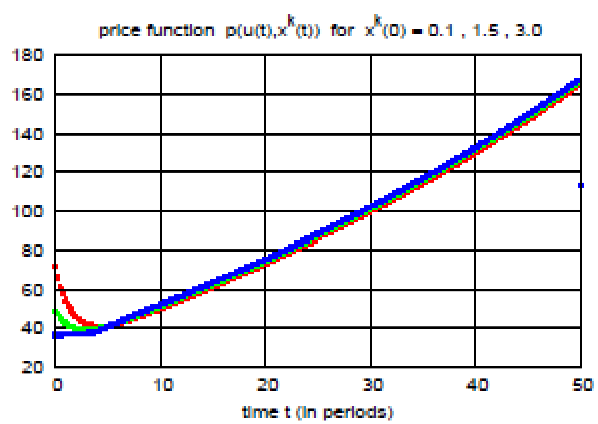
IV.2. A Model: Oil Discovery and Extraction–Finite Horizon

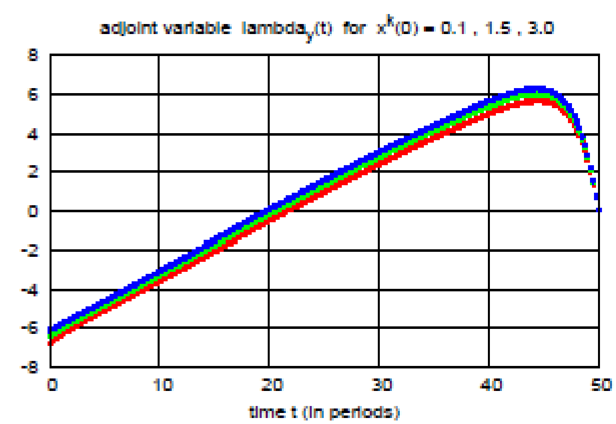
$$p(u, x^k, y) = \left(\frac{1}{\gamma + w_u \cdot u + w_x \cdot x^k - \mu y}\right)^{\alpha}$$

With finite horizion: Case $T = 50$, $w_x = 0.005$, $\mu = 0.002$ $\quad w_u = 1$





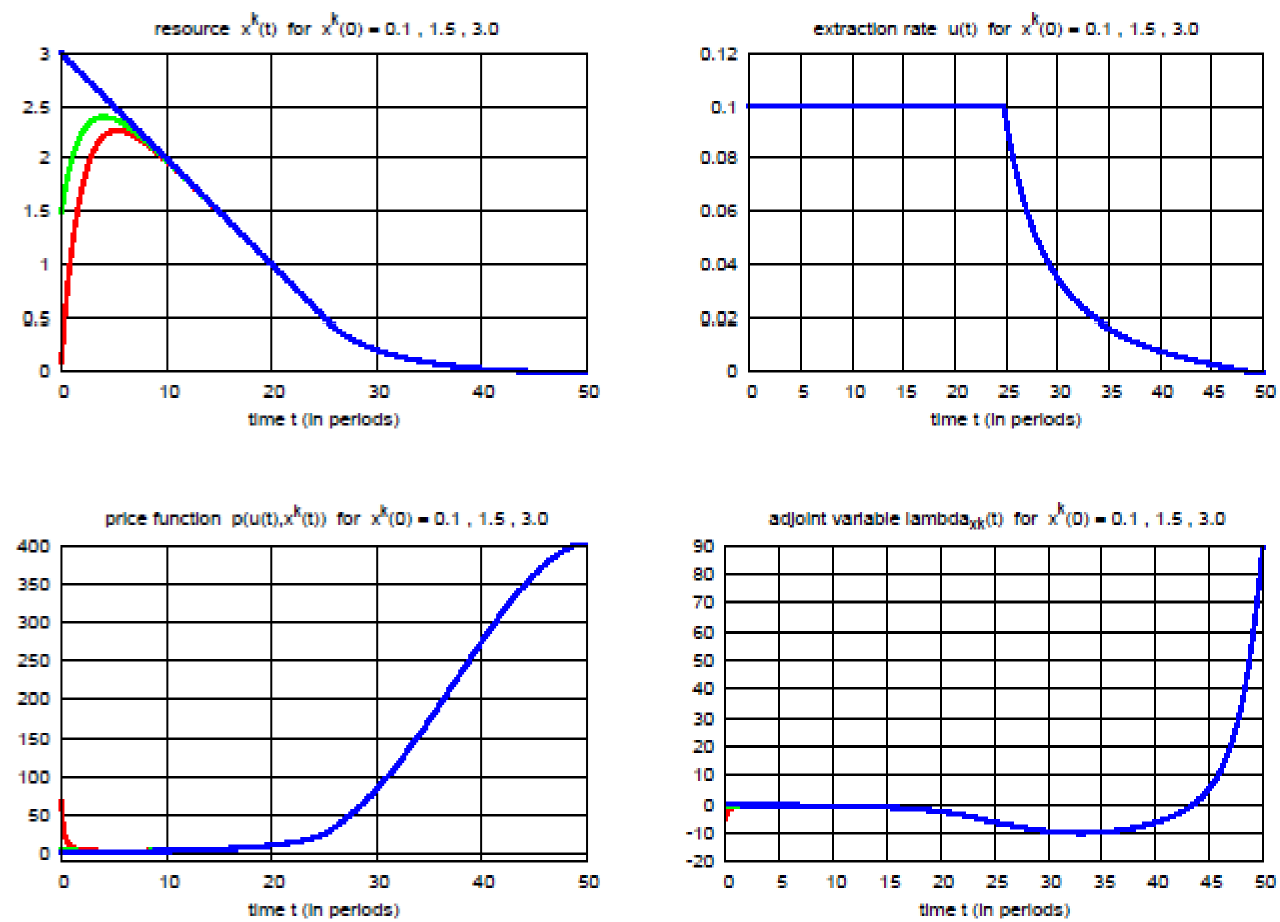




IV.2. A Model: Oil Discovery and Extraction–Finite Horizon

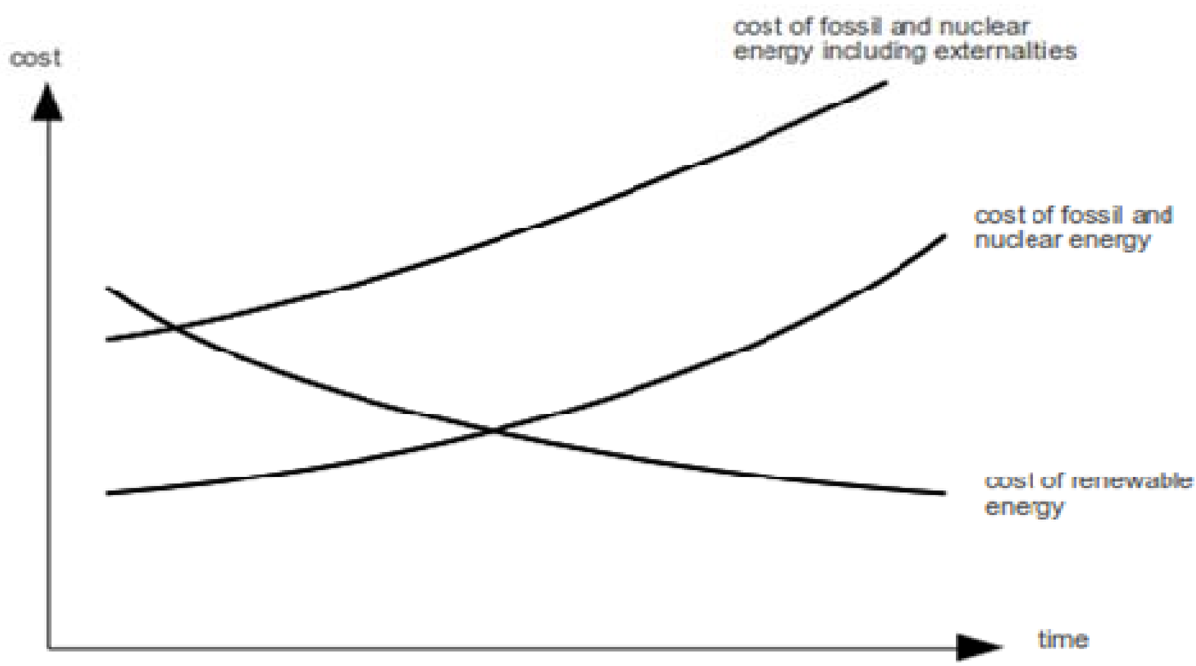
$$p(u, x^k, y) = \left(\frac{1}{\gamma + w_u \cdot u + w_x \cdot x^k - \mu y}\right)^{\alpha}$$

With finite horizion: Case $T = 50,\ w_u = 0.5,\ w_x = 0.2,\ \mu = 0$



V. Fossil and Renewable Energy in an Optimal Growth Model (Data Source: IEA 2011)

Back to Empirical Results: Long run cost trends of Fossil Energy (including externalities) and Renewable Energy
http://www.economicpolicyresearch.org/economics-of-climate-change.html



V. Fossil and Renewable Energy in an Optimal Growth Model

(Greiner and Semmler, 2011)

Growth model with **externalties** from fossil and renewable energy:

$$E_f = A_f u,\ E_r = A_r K,$$
$$Y = AE^{\alpha} = A(A_r K + A_f u)^{\alpha}$$

$$Max_{\{C,u\}} \quad \int_0^{\infty} e^{-rt} \left(\frac{C^{1-\sigma}(M - M_o)^{-\xi(1-\sigma)} - 1}{1-\sigma} \right) dt$$

s. t. $\dot{K} = Y - C - \delta K - a \cdot u$ (Market solution: $-\tau u - \Gamma + \theta \dot{K}$)

$\dot{R} = -u$ (or alternatively: $\dot{R} = \alpha_1 R(\bar{R} - R) - u$)

$$\dot{M} = \beta_1 u - \mu(M - \kappa M_o)$$

$$R(0) = R_0, K(0) = K_0, M(0) = M_0 \geq M_o \text{ given}$$

$$\lim_{t\to\infty} e^{-rt} K(t) \geq 0, \lim_{t\to\infty} R(t) \geq 0, \lim_{t\to\infty} M(t) \geq M_o$$

VI. Conclusions

- Partial Model with fossil energy—how do prices move and does fossil fuel remain unextracted?

  -- infinite horizon

  -- finite horizon

We currently explore extensions of this type of model, including (further work with H. Maurer):

-- delays for control or state equations

-- discovery rate as control

- We have indicated how to build in fossil and renewable energy into optimal growth model with negative externality. Next step: to explore carbon tax and subsidy to renewable energy in a market economy, and the fate of R?

Appendix 2: Speculation

- Eric Tham (2008), „Time Varying Factors behind the Oil Price“