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science for global insight

Optimal Resource Allocation (3)

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Part 3-1:

Modularized nexus modeling

What is a cost-efficient approach to model food-water-energy nexus?

- Capitalize on already existing sectorial and regional model
- Modular approach might be an effective way to integrate/link sectorial and regional models

- As an alternative to creating one joint model

Model A

$$c^A x^A \rightarrow \min_{x^A}$$

$$D^A x^A \geq d^A$$

$$R^A x^A \leq r^A$$

$$x^A \geq 0$$

Minimizing costs

Constraints on volume of production

Constraints on available resources (land, water, etc.)

Model E

$$c^E x^E \rightarrow \min_{x^E}$$

$$D^E x^E \geq d^E$$

$$R^E x^E \leq r^E$$

$$x^E \geq 0$$

Conceptual idea of linking models

Model A

$$c^A x^A \to \min_{x^A}$$

$$D^A x^A \ge d^A$$

$$R^A x^A \le r^A$$

$$x^A \ge 0$$

☹

Minimizing net costs

Constraints on volume of production

Constraints on available resources (land, water, etc.)

Separation is artificial!

Model E

$$c^E x^E \to \min_{x^E}$$

$$D^E x^E \ge d^E$$

$$R^E x^E \le r^E$$

$$x^E \ge 0$$

☹

$$R^A y^A + R^E y^E \le r \qquad (r^A + r^E = r)$$

☺ ☺ ☺

Joint constraint linking model A and model E

How to organize the model linkage?

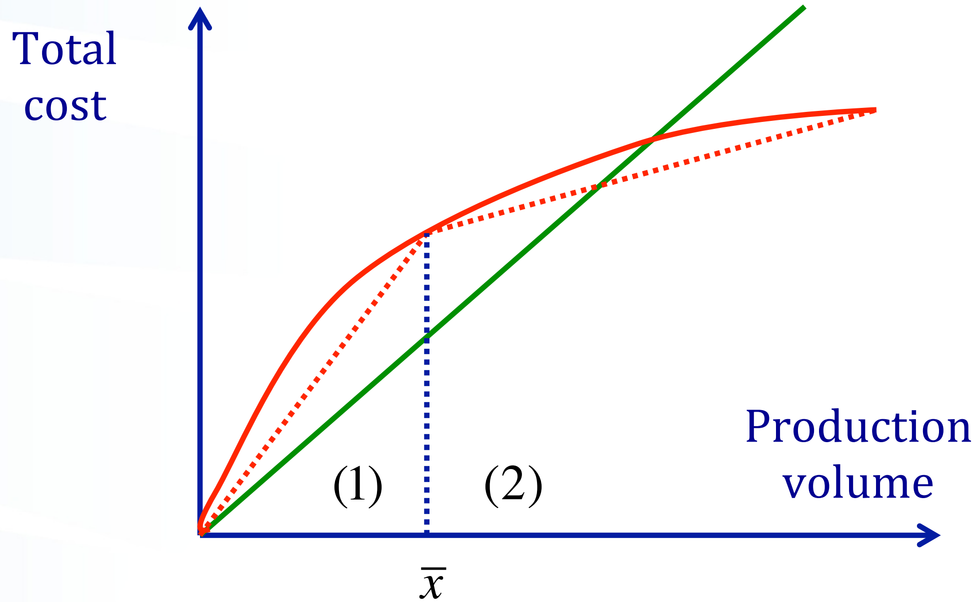
- Generalized Nash Equilibrium (G Debreu, 1952)
  - Iterative exchange of resource "quotas" between models until the process converges
  - Existence and uniqueness is not guaranteed
  - Convergence is not guaranteed
- Hard integration
  - Single objective function – socially optimal solutions
  - Requires different modeling teams working together
  - The code can be gigantic
- Decentralized integration (Ermoliev, 1980 - IIASA)
  - Iterative algorithm of updating "quotas" re-calculated by a "central hub" relying on dual variables
  - Converges to the socially optimal solution

Questions?

Part 3-2:
Non-linearities

Example: Effects of the economy of scale in the two-crop model

$$
\begin{aligned}
& c_A x_A + c_B x_B \to \min \\
& w_A x_A + w_B x_B \le w \\
& x_A + x_B \ge D \\
& x_A \ge 0 \\
& x_B \ge 0
\end{aligned}
$$



For example: Solve two problems and choose a solution that is consistent

$$
\begin{aligned}
& c_A^{(1)} x_A + c_B^{(1)} x_B \to \min \\
& w_A x_A + w_B x_B \le w \\
& x_A + x_B \ge D \\
& 0 \le x_A \le \bar{x} \\
& 0 \le x_B \le \bar{x}
\end{aligned}
$$

$$
\begin{aligned}
& c_A^{(2)} x_A + c_B^{(2)} x_B \to \min \\
& w_A x_A + w_B x_B \le w \\
& x_A + x_B \ge D \\
& x_A \ge \bar{x} \\
& x_B \ge \bar{x}
\end{aligned}
$$

Questions?

Part 3-3:
Dynamics

Example: Dynamics in the two-crop model

$$c_A x_A + c_B x_B \to \min$$
$$w_A x_A + w_B x_B \le w$$
$$x_A + x_B \ge D$$
$$x_A \ge 0$$
$$x_B \ge 0$$

For one year

For two years

$$c_A(1)x_A(1) + c_B(1)x_B(1) + c_A(2)x_A(2) + c_B(2)x_B(2) \to \min$$
$$w_A(1)x_A(1) + w_B(1)x_B(1) \le w(1)$$
$$w_A(2)x_A(2) + w_B(2)x_B(2) \le w(2)$$
$$x_A(1) + x_B(1) \ge D(1)$$
$$x_A(2) + x_B(2) \ge D(2)$$
$$x_A(1) \ge 0, x_A(2) \ge 0$$
$$x_B(1) \ge 0, x_B(2) \ge 0$$

Example: Dynamics in the two-crop model

For N years

$$\sum_{t=1}^{N}\left(c_A(t)x_A(t)+c_B(t)x_B(t)\right)\to \min$$

$$w_A(t)x_A(t)+w_B(t)x_B(t)\le w(t)\quad \forall t=1,\dots,N$$

$$x_A(t)+x_B(t)\ge D(t)\quad \forall t=1,\dots,N$$

$$x_A(t)\ge 0\quad \forall t=1,\dots,N$$

$$x_B(t)\ge 0\quad \forall t=1,\dots,N$$

Discount factor: Future is less important than the present time!

$$\sum_{t=1}^{N}\left(\frac{1}{1+\rho}\right)^{t}\left(c_A(t)x_A(t)+c_B(t)x_B(t)\right)\to \min \qquad \rho\approx 1-5\%$$

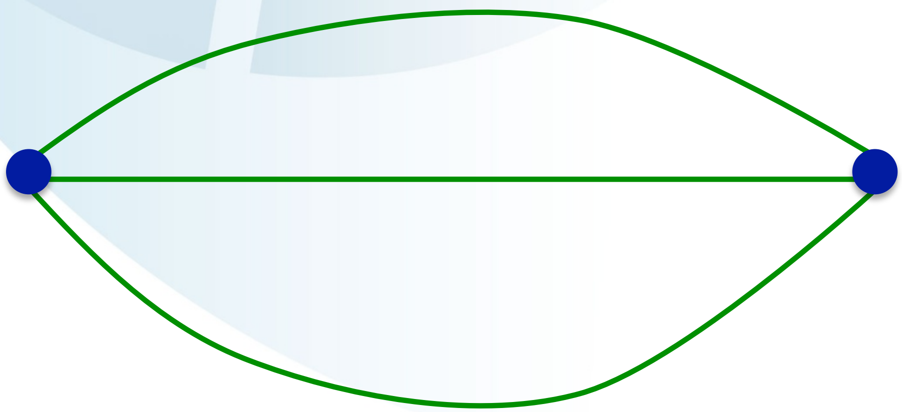
More advanced way of considering dynamic optimization: Optimal control

$$\int_0^T F(t, x(t), u(t))\,dt \to \min_{u(\cdot)}$$

$$\dot{x}(t) = g(t, x(t), u(t))$$

$$x(0) = x_0, x(T) = x_T$$

$$u(t) \in U$$



Simple application: How to move from point A to point B with minimal effort?

Founders of the optimal control methodology

Lev Pontryagin
Maximum principle

Richard Bellman
Dynamic programming

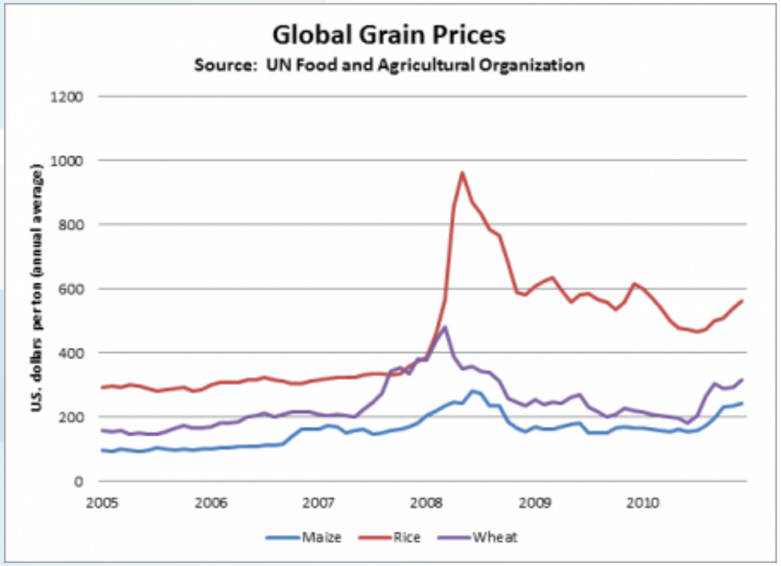
Applications initially focused on military and space problems, but currently extend to economic and environmental problems

Questions?

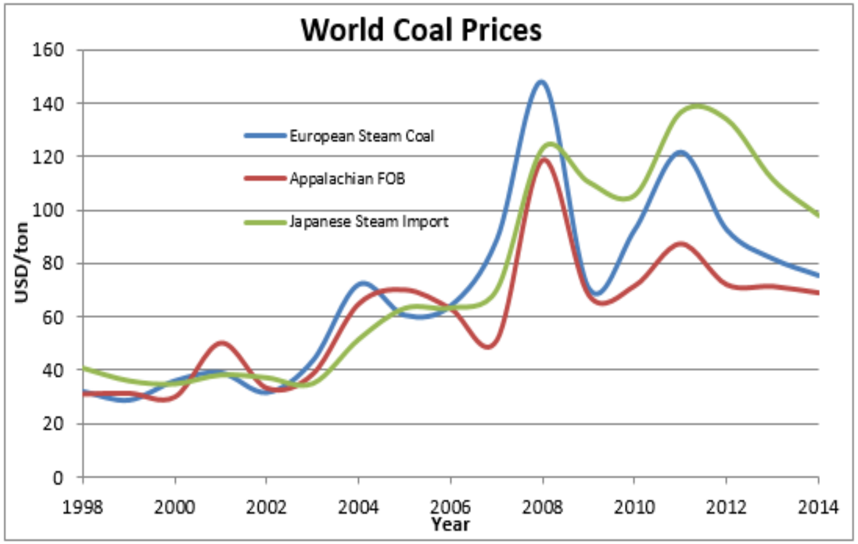
Part 3-4:
Stochastic optimization

Resource allocation problems are subject to various uncertainties

Volatility of prices on crops



Volatility of prices on energy carriers



- Natural factors – weather, rainfalls, water discharge in rivers etc. is also highly uncertain
- To enable more robust planning, one should account for uncertainty

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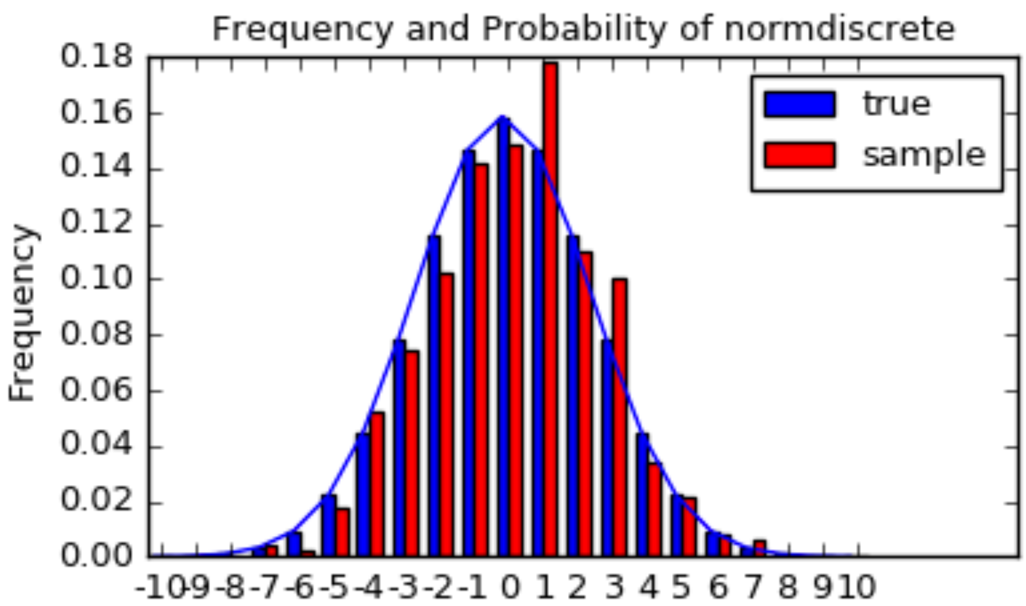
How to represent uncertainty mathematically?

- Probabilistic distributions



How to include the uncertainty in LP models?

- There are various ways, technically advanced – stochastic optimization
- Min-max approach is arguably the simplest way – optimize the worst outcome – guaranteed control

$$F(x,p) \to \max_x$$

$$x \in X$$

$$\Downarrow$$

$$x^*(p)$$

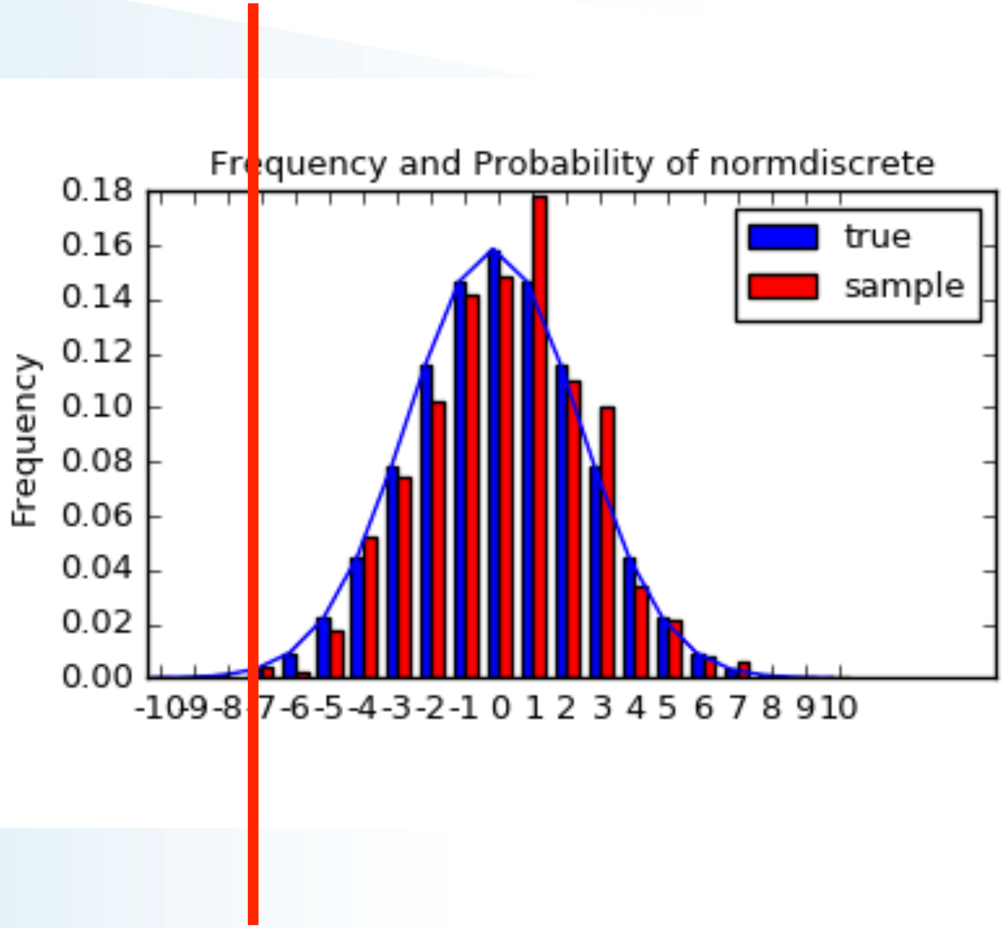
$$F(x,p) \to \min_p \max_x$$

$$x \in X, p \in P$$

$$\Downarrow$$

$$x^*$$

A stochastic version of the min-max approach



- Cut the tail of the distribution at some level (e.g., 5%)
- Replace the "min" part of the optimization over p

Questions?

Young Scientists Summer Program at IIASA

- June, July, August each year
- A research project in collaboration with IIASA scientists
- Rich scientific and social program
- ~50 international participants from all over the world
- Enrollment in a PhD program is required
- Questions: Pakistani NMO

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Thank you
for your attention!

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