Scientific Program

13^{th} Viennese Workshop on Optimal Control and Dynamic Games

Tuesday 21st April, 2015

Session numbers and organizers are written in white

		Prechtlsaal I	Prechtlsaal II	Festsaal	Böcklsaal
Wed	Session 4	5 Baier, Farkhi	6 Boucekkine	7	
May, 13	11:30-12:45	Kalman	Rakovic	Growec	Dilmaghani
		Kurzhanski	Junge	Privileggi	Orlov
		Zelikin	Mokhov	Boucekkine	Skritek
	Session 8	Chibulka	9 Neitzel, Rösch	10 Wrzaczek	11
	14:00-15:40	Ioffe	Wollner	Cabo	De Giovanni
		Dontchev	Apel	Gromov	de Frutos Cachorro
		Frankowska	De los Reyes	Reddy	Rodriguez
		Fabian	Hintermüller	Wrzaczek	Fruchter
	Session 12	Donato, Milasi	13 Baier, Farkhi	14 Kuhn, Wrzaczek	15 Lambertini
	16:00-17:15	Rockafellar	Rieger	Strulik	Lambertini
		Donato	Baier	Heijdra	El Ouardighi
		Krastanov	Lygeros	Kuhn	Mantovani
	Session 16	Donato, Milasi	17 Baier, Farkhi	18 Prettner, Prskawetz	19
	17:30-18:45	Aussel	Riedl	Bucci	Cheung
		Milasi	Novikova	Werner	Engwerda
		Giuffre	Widder	Schaefer	Kodritsch

Thu	Session	21 Cibulka	22 Falcone, Zidani	23 Prettner, Prskawetz	24 Zaccour
May, 14	10:00-11:15	Dreves	Picarelli	Perez-Sebastian	Doyen
		Strugariu	Bouchard	d'Albis	Sorger
		Cervinka	Cacace	Sanchez-Romero	de Frutos
	Session	25 Cibulka	26 Zaslavski	27 Kuhn, Wrzaczek	28 Zaccour
	11:30-12:45	Cibulka	Aseev	Ludwig	Petrosyan
		Tonon	Bachir	Schneider	Marin-Solano
		Ribarska	Blot	Dragone	Gromova
	Session	29 Faggian, Gozzi	30 Zaslavski	31 Boucekkine	32
	14:00-15:40	Frederico	Maruyama	Laffargue	Chen
		Prosdocimi	Hayek	Bluescke	Kogan
		De Angelis	Hosoya	Palokangas	Kopel
		Bandini	Zemel	Zou	Lozovanu
	Session	33 Faggian, Gozzi	34 Neitzel, Rösch	35 Semmler	36 Behringer, Upmann
	16:00-17:40	Boucekkine	Chrysafinos	Bondarev	Schubert
		Manfredi	Ryll	Schwerin	Xabadia
		Di Girolami	Jordan	Pavlova	Anita
		Hota	Kimmerle	Semmler	Benchekroun

Fri	Session	38 Faggian, Gozzi	39 Albi, Fornasier	40 Camacho, Saglam	41 Falcone, Zidani
May, 15	10:00-11:15	Bambi	Festa	Perez-Barahona	Cirant
		Behringer	Bensoussan	Zou	Ferretti
		Gozzi	Di Francesco	Schumacher	Reisinger
	Session	42 Faggian, Gozzi	43 Albi, Fornasier	44 Wagener	45
	11:30-12:45	Freni	Goatin	Altaghlibi	Apreutesei
		Belyakov	Herty	Kiseleva	Bogusz
		Dmitruk	Martin	Heijnen	Uecker
	Session	46 Alt, Felgenhauer	47 Zaslavski	48 Wagener	49 Upmann, Behringer
	14:00-15:40	Coletsos	Khlopin	Rowat	Camacho
		Bagagiolo	Tsur	Keoula	Rosestolato
		Felgenhauer	Yu	Wang	Upmann
		Gerdts	Zaslavski	Zeppini	Yannacopoulos
	Session	50 Faggian, Gozzi	51 Albi, Fornasier	52 Kort	53 Feichtinger, Seidl, Tragler
	16:00-17:40	Swiech	Solombrino	Hagspiel	Seidl
		Masiero	Wolfram	Huberts	Kraszewksa
		Zanco	Kalise	Lavrutich	Novak
		Cosso	Bongini	Thijssen	Feichtinger

Sat	Session	55 Alt, Felgenhauer	56 Neitzel, Rösch	57 Kort	58
May, 16	09:55-11:35	Korytowski	Pfeiffer	Nishihara	Martin-Herran
		Kostina	Neitzel	Vermeulen	Wirl
		Rund	Rösch	Wen	Krawczyk
		Poggiolini	Schiela	Compernolle	Zimka
	Session	59 Alt, Felgenhauer	60 Zaslavski	61 Camacho, Saglam	62 Zaccour
	11:45-13:00	Schneider	Gaitsgory	Augeraud-Veron	De Giovanni
		Seydenschwanz	Kurina	Esfahani	Jorgensen
		Strelkovski	Rampazzo	Drugeon	Zaccour

Wednesday, May 13

1 Opening **08:45 – 09:10**
Kuppelsaal

2 Plenary **09:10 –10:00**
***Chair*: Session Chair** **Kuppelsaal**

3 Plenary **10:00 –10:50**
***Chair*: Session Chair** **Kuppelsaal**

Coffee break

Poster presentations **10:50 –11:30**
Kuppelsaal

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4 Systems and control **11:30 – 12:45**
***Chair*: S. Chair** **Prechtlsaal I**

5 Set-valued numerics and control 1 **11:30 – 12:45**
***Chair*: R. Baier, E. Farkhi** **Prechtlsaal II**

6 Transitional dynamics in endogenous growth models 1 **11:30 – 12:45**
***Chair*: R. Boucekkine** **Festsaal**

7 Economic applications of optimal control 1 **11:30 – 12:45**
***Chair*: S. Chair** **Böcklsaal**

Lunch Break

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8 Set-valued and variational analysis 1 **14:00 – 15:40**

***Chair*: R. Cibulka** **Prechtlsaal I**

9 PDE-constrained optimization 1 **14:00 – 15:40**

***Chair*: I. Neitzel, A. Rösch** **Prechtlsaal II**

10 Regime switching in differential games **14:00 – 15:40**

***Chair*: S. Wrzaczek** **Festsaal**

11 Resource economics **14:00 – 15:40**

***Chair*: Session Chair** **Böcklsaal**

Coffee Break

12 Variational inequalities and applications in economics 1 **16:00 – 17:15**

***Chair*: M. Donato, M. Milasi,** **Prechtlsaal I**

13 Set-valued numerics and control 2 **16:00 – 17:15**

***Chair*: R. Baier, E. Farkhi** **Prechtlsaal II**

14 Health care 1 **16:00 – 17:15**

***Chair*: M. Kuhn, S. Wrzaczek** **Festsaal**

15 Pollution abatement in dynamic oligopoly games **16:00 – 17:15**

***Chair*: L. Lambertini** **Böcklsaal**

Break

Thursday, May 14

20 Plenary **08:45 – 09:35**

***Chair*: Session Chair** **Festsaal**

Coffee break

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21 Set-valued and variational analysis 2 **10:00 – 11:15**

***Chair*: R. Cibulka** **Prechtlsaal I**

22 Stochastic control and applications 1 **10:00 – 11:15**

***Chair*: M. Falcone, H. Zidani** **Prechtlsaal II**

23 Economic growth and demographic change 2 **10:00 – 11:15**

***Chair*: K. Prettner, A. Prskawetz** **Festsaal**

24 Differential games in environmental economics **10:00 – 11:15**

***Chair*: G. Zaccour** **Böcklsaal**

Break

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25 Set-valued and variational analysis 3 **11:30 – 12:45**

***Chair*: R. Cibulka** **Prechtlsaal I**

26 Infinite horizon optimal control and mathematical economics 1 **11:30 – 12:45**

***Chair*: A. Zaslavski** **Prechtlsaal II**

27 Health care 2 **11:30 – 12:45**

***Chair*: M. Kuhn, S. Wrzaczek** **Festsaal**

28 Cooperation in differential games **11:30 – 12:45**

***Chair*: G. Zaccour** **Böcklsaal**

Lunch Break

29 Optimal control in finite and infinite dimension and application to economics 1

14:00 – 15:40

***Chair*: S. Faggian, F. Gozzi** **Prechtlsaal I**

30 Infinite horizon optimal control and mathematical economics 2 **14:00 – 15:40**

***Chair*: A. Zaslavski** **Prechtlsaal II**

31 Dynamic games models of institutional change **14:00 – 15:40**

***Chair*: R. Boucekkine** **Festsaal**

32 Dynamic games in economics and business 2 **14:00 – 15:40**

***Chair*: S. Chair** **Böcklsaal**

Coffee Break

33 Optimal control in finite and infinite dimension and application to economics 2

16:00 – 17:40

***Chair*: S. Faggian, F. Gozzi** **Prechtlsaal I**

34 PDE-constrained optimization 2 **16:00 – 17:40**

***Chair*: I. Neitzel, A. Rösch** **Prechtlsaal II**

35 Economics of climate change **16:00 – 17:40**

***Chair*: W. Semmler** **Festsaal**

36 Harvesting 1 **16:00 – 17:40**

***Chair*: S. Behringer, T. Upmann** **Böcklsaal**

Friday, May 15

37 Plenary **08:45 – 09:35**

***Chair*: Session Chair** **Festsaal**

Coffee break

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38 Optimal control in finite and infinite dimension and application to economics 3

10:00 – 11:15

***Chair*: S. Faggian, F. Gozzi** **Prechtlsaal I**

39 Mean-field models and control of multi-agent systems 1 **10:00 – 11:15**

***Chair*: G. Albi, M. Fornasier** **Prechtlsaal II**

40 Growing economies and the environment **10:00 – 11:15**

***Chair*: M.C. Camacho, C. Saglam** **Festsaal**

41 Stochastic control and applications 2 **10:00 – 11:15**

***Chair*: M. Falcone, H. Zidani** **Böcklsaal**

Break

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42 Optimal control in finite and infinite dimension and application to economics 4

11:30 – 12:45

***Chair*: S. Faggian, F. Gozzi** **Prechtlsaal I**

43 Mean-field models and control of multi-agent systems 2 **11:30 – 12:45**

***Chair*: G. Albi, M. Fornasier** **Prechtlsaal II**

44 Optimal management in complex environments 1 **11:30 – 12:45**

***Chair*: F. Wagener** **Festsaal**

45 Distributed control in economics and management **11:30 – 12:45**

***Chair*: Session Chair** **Böcklsaal**

Lunch Break

46 Stability and numerical methods for optimal control 1 **14:00 – 15:40**

***Chair*: W. Alt, U. Felgenhauer** **Prechtlsaal I**

47 Infinite horizon optimal control and mathematical economics 3 **14:00 – 15:40**

***Chair*: A. Zaslavski** **Prechtlsaal II**

48 Optimal management in complex environments 2 **14:00 – 15:40**

***Chair*: F. Wagener** **Festsaal**

49 Harvesting 2 **14:00 – 15:40**

***Chair*: S. Behringer, T. Upmann** **Böcklsaal**

Coffee Break

50 Optimal control in finite and infinite dimension and application to economics 5

16:00 – 17:40

***Chair*: S. Faggian, F. Gozzi** **Prechtlsaal I**

51 Mean-field models and control of multi-agent systems 3 **16:00 – 17:40**

***Chair*: G. Albi, M. Fornasier** **Prechtlsaal II**

52 Real options 1 **16:00 – 17:40**

***Chair*: P.M. Kort** **Festsaal**

53 Economics of crime **16:00 – 17:40**

***Chair*: G. Feichtinger, A. Seidl, G. Tragler** **Böcklsaal**

Saturday, May 16

54 Plenary **08:45 – 09:35**
***Chair*: Session Chair** **Festsaal**

Coffee break

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55 Stability and numerical methods for optimal control 2 **09:55 – 11:35**
***Chair*: W. Alt, U. Felgenhauer** **Prechtlsaal I**

56 PDE-constrained optimization 3 **09:55 – 11:35**
***Chair*: I. Neitzel, A. Rösch** **Prechtlsaal II**

57 Real options 2 **09:55 – 11:35**
***Chair*: P.M. Kort** **Festsaal**

58 Economic applications of optimal control 2 **09:55 – 11:35**
***Chair*: S. Chair** **Böcklsaal**

Break

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59 Stability and numerical methods for optimal control 3 **11:45 – 13:00**
***Chair*: W. Alt, U. Felgenhauer** **Prechtlsaal I**

60 Singularities and optimality conditions in control theory **11:45 – 13:00**
***Chair*: A. Zaslavski** **Prechtlsaal II**

61 Non-monotonic dynamics in economics **11:45 – 13:00**
***Chair*: M.C. Camacho, C. Saglam** **Festsaal**

62 Differential games in marketing and operations **11:45 – 13:00**
***Chair*: G. Zaccour** **Böcklsaal**

Poster presentations

Wednesday, May 13

1. Opening **08:45 – 09:10**

Kuppelsaal

2. Plenary **09:10 –10:00**

***Chair*: Session Chair** **Kuppelsaal**

Sustainability of cooperation in a class of stochastic games

Georges Zaccour (1),

(1) GERAD, HEC Montréal, Montréal, Canada

A well-known problem in dynamic cooperative games is the sustainability of cooperation over time. The literature addressed this issue following different approaches, namely, the design of time-consistent payments, incentive equilibrium strategies and trigger strategies that deter credibly and effectively deviation from cooperation.

In this talk, I will apply these different approaches to dynamic games played over event trees, that is, stochastic games where the uncertainty is not influenced by players' actions but it is nature's decision. After introducing the main elements of this class of games, I will introduce node-consistent cooperative payments based on the Shapley value and imputations in the core as means for sustaining cooperation over nodes (and time). Further, I will show how incentive and trigger strategies can be constructed to strategically support the cooperative agreement designed at the starting date of the game.

3. Plenary **10:00 –10:50**

***Chair*: Session Chair** **Kuppelsaal**

Dynamic stability of the Nash equilibrium for a bidding game

Alberto Bressan (1),

(1) Department of Mathematics, Penn State University, USA

A one-sided limit order book is modeled as a noncooperative game for several players. An external buyer asks for an amount $X > 0$ of a given asset. This amount will be bought at the lowest available price, as long as the price does not exceed a upper bound P. One or more sellers offer various quantities of the asset at different prices, competing to fulfill the incoming order. The size X of the order and the maximum acceptable price P are not a priori known, and are thus regarded as random variables. In this setting, a unique Nash equilibrium exists, where each seller optimally prices his assets in order to maximize his own expected profit.

A dynamics can be introduced, assuming that each player gradually adjusts his pricing strategy in reply to the strategies adopted by all other players. In the case of (i) infinitely many small

players or (ii) two large players with one dominating the other, we show that the pricing strategies asymptotically converge to the Nash equilibrium.

4. Systems and control — 11:30 – 12:45

***Chair*: S. Chair** — **Prechtlsaal I**

Insight from the outside

Rudolf Kalman (1,2),

(1) University of Florida (2) ETH Zürich

In any control problem the first step toward a solution requires knowledge of the dynamics of the object to be controlled. When that object is known only through its behavior (frequency response or input/output relation), our information about the internal structure of the object is limited to the completely observable/completely controllable subsystem—well known for more than 50 years, this result is about the maximum achievable by linear mathematics.

The situation is different when the dynamical object is composed of finitely many elements—as is the case of (i) passive electrical networks, composed of resistors, capacitors, and inductors or (ii) atoms composed of neutrons, electrons, and protons. Here classical algebraic invariant theory (Hilbert) gives much more information about the internal structure of such systems. For example, invariants determine when networks are equivalent, or in case of atoms, isotopes.

Invariant theory is (nonlinear) mathematics that will greatly deeply enrich, and has already enriched, our knowledge of systems.

The problem of collision avoidance in group control

Alexander B. Kurzhanski (1),

(1) Faculty of Computational Mathematics and Cybernetics, Lomonosov Moscow State University, Russia

The problems considered in this report are on designing coordinated feedback strategies for groups (teams) of controlled motions which arise in many applied areas. An array of systems is a group (a team)if throughout the motion its members should be not too close, avoiding collisions, while also not to far from each other, remaining within a preassigned virtual ellipsoidal tube (the “container” of the team). The motion of each team-mate is governed by second order dynamics for a trajectory that lies at the center of a ball of given “safety radius”. Collisions would then be avoided once the system controls would ensure that the interiors of such balls would never intersect with each other. This report focuses on mathematical solution of such problem which is to ensure collision avoidance throughout the motion, while remaining within the virtual container. The is reflected in treating an array of convex and complementary convex state constraints. The problem is solved in finite time by using the techniques of nonlinear analysis within a general Hamilton-Jacobi-Bellman framework.

Generic fractal structure of optimal synthesis in problems with affine multidimensional control

Mikhail I. Zelikin (1), Roland Hildebrand (2), Lev V. Lokutsievskiy (1),

(1) Institute of Mathematics and Mechanics, Lomonosov Moscow State University, Russia
(2) Weierstrass Institute for Applied Analysis and Stochastics

It is designed the phase portrait of optimal synthesis (the geometric behavior of the set of optimal trajectories) for a model problem with control taking values in a two-dimensional simplex, having singular extremals of the second order and modes of infinite accumulations of switches. The new phenomenon is the chaotic behavior of the set of trajectories of optimal synthesis. The set of optimal non-wandering trajectories (NW) has the structure of Cantor's set. The dynamics of the system is described by a topological Markov chain. The entropy and the Hausdorf dimension of the set NW is calculated. It is shown that similar behavior is generic for piecewise smooth Hamiltonian systems in the vicinity of the intersection of three codimension 1 break-surface strata.

5. Set-valued numerics and control 1 — 11:30 – 12:45

***Chair*: R. Baier, E. Farkhi** — **PrechtlsaaI II**

Model predictive control with generalized terminal conditions

Saša V. Raković (1), William S. Levine (2),

(1) Independent Researcher, London, UK (2) Institute for Systems Research, The University of Maryland, College Park, USA

Model predictive control (MPC) is best summarized as a repetitive decision making process in which the underlying decision making takes the form of an open–loop, finite horizon, optimal control (OC). MPC induces positive invariance and stability under relatively mild conditions on the problem setup. The chief components of these conditions are either the introduction of terminal constraint set and cost function, or the utilization of a sufficiently long horizon length. In either case, MPC provides a sensible approximation to a highly desirable infinite horizon OC. However, the conditions on terminal constraint set and cost function, or on horizon length, are global in their nature and, thus, independent of the current state. These facts highlight crucial weaknesses of the MPC approaches.

We offer MPC with generalized terminal conditions. In particular, we propose the utilization of a terminal constraint set and cost function that are allowed to depend on the current state. In turn, this leads to an improved MPC with the potential to provide strictly finer approximation and, from a theoretical point of view, even the exact solution to infinite horizon OC problem. We also propose the use of terminal constraint sets and cost functions generated by a suitably defined set and functional dynamics. For the latter proposal, we discuss set and functional dynamics of terminal constraint sets and cost functions, respectively, in a general setting. Furthermore, motivated by underlying intricate numerical aspects, we also explore restrictions of these dynamics to particular families of terminal constraint sets and cost functions. Finally, we demonstrate that

for some special, but frequently encountered, instances our proposal allows for an improved MPC at a negligibly increased computational cost.

Dynamic programming using radial basis functions

Oliver Junge (1), Alex Schreiber (1),

(1) Center for Mathematics, Technische Universität München, Garching, Germany

We propose a discretization of the optimality principle in dynamic programming based on radial basis functions and Shepard's moving least squares approximation method. We prove convergence of the value iteration scheme, derive a statement about the stability region of the closed loop system using the corresponding approximate optimal feedback law and present several numerical experiments.

Representations of multifunctions by metric selections and the metric set-valued integral

Alona Mokhov (1, 2), Nira Dyn (2), Elza Farkhi (2),

(1) Afeka, Tel-Aviv Academic College of Engineering, Tel Aviv, Israel (2) Tel-Aviv University, Tel Aviv, Israel

In this work we consider representations of set-valued functions (SVFs) with compact images in R^n by special selections through each point of the graph. Using representations by selections, the approximation of the set-valued function is reduced to the approximation of the corresponding collection of representing single-valued functions. Thus well-known tools from constructive function theory can be applied directly. The main effort here is the design of an appropriate representation consisting of single-valued functions with regularity properties "inherited" from those of the approximated SVF. The case of convex-valued multifunction with the support functions is well-developed. The case of SVFs with general images is less studied. We introduce a representation of a multifunction by specific *metric* selections with low variation and with regularity properties as those of the set-valued function. We also introduce a new notion of set-valued integral based on the metric selections. This new *metric* integral is different from the classical one - the Aumann integral, and may be nonconvex. We study some properties of the metric integral and give examples.

6. Transitional dynamics in endogenous growth models 1 — 11:30 – 12:45

***Chair*: R. Boucekkine** — **Festsaal**

Endogenous labor share cycles: theory and evidence

Jakub Growiec (1,2), Peter McAdam (3,4), Jakub Mućk (1,2),

(1) Warsaw School of Economics, Poland (2) Narodowy Bank Polski (3) European Central Bank (4) University of Surrey

Based on long US time series we document a range of empirical properties of the labor's share of GDP, including its substantial medium-run swings. We explore the extent to which these empirical regularities can be explained by a calibrated micro-founded long-run economic growth model with normalized CES technology and endogenous labor- and capital-augmenting technical change driven by purposeful directed R&D investments. It is found that dynamic macroeconomic trade-offs created by arrivals of both types of new technologies may lead to prolonged swings in the labor share due to oscillatory convergence to the balanced growth path as well as stable limit cycles via Hopf bifurcations. Both predictions are broadly in line with the empirical evidence.

A competitive idea-based growth model with shrinking workers' income share

Fabio Privileggi (1), Carla Marchese (2),

(1) Dept. of Economics and Statistics "Cognetti de Martiis", University of Torino, Torino, Italy (2) Institute POLIS - DiGSPES, University of Piemonte Orientale "Amedeo Avogadro", Alessandria, Italy

In this paper we present a model in which endogenous growth arises in competitive markets. Knowledge is described as a labor-augmenting factor used directly in the final goods' production. Firms demand both basic nonrival knowledge contents, which are supplied jointly and inelastically with raw labor, and further contents supplied by patent holders. This fact, together with Lindahl prices for knowledge, allows competition to work, while it also implies that workers' income share declines overtime. In a first version of the model with constant cost of knowledge production the first best is attained. In further versions of the model, in which the cost of knowledge production is allowed to change over time and externalities arise, in a decentralized economy a second best equilibrium occurs in the transitional period, while in the long run there is convergence to efficiency. As the asymptotic equilibrium exhibits strong scale effects, we propose a final version of the model with only weak scale effects under the assumption that combining labor and knowledge becomes increasingly difficult.

A Lipsetian theory of institutional change

Raouf Boucekkine (1), Paolo Piacquadio (2), Fabien Prieur (3),

(1) Aix-Marseille School of Economics, Marseille, France (2) ESOP and Department of Economics, UiO, Oslo, Norway (3) INRA and University of Montpellier I, Montpellier, France

We study the paradigmatic case of a dictator with full political and economic control. The dictator has 3 main controls. First, he is aware of a revolutionary threat and acts in such a way that a revolution will never occur through redistribution. This threshold does depend on the education level of population: the larger the human capital of citizens, the higher their income and consumption aspirations, and the more costly redistribution. Education policy is the second instrument in the hands of the dictator. Human capital is a production factor (Cobb-Douglas technology. The last control is institutional: in contrast to the recent literature on democratization games, the decision to stop dictatorship is not taken by citizens via a costly revolution (since the

revolution threat is fully internalized) but by the dictator himself. Our theory is compatible with Lipset thought in the three essentiel dimensions: the link human capital-insitutional change, the link income-insitutional change and, more remotely, the link inequality-institutional change.

7. Economic applications of optimal control 1 — 11:30 – 12:45

***Chair*: S. Chair** — **Böcklsaal**

Revisiting sacrifice and stigma: Why do older churches become more liberal?

Maryam Dilmaghani (1),

(1) Department of Economics, Sobey School of Business, Saint Mary's University, Halifax, Canada

The paper revisits the concept of "Sacrifice and Stigma" first introduced in the seminal paper of Iannaccone (1992). Iannaccone provides an explanation for the rise of strict churches compared to more liberal ones, which relies on their requirement of "sacrifice and stigma". In his conception, "sacrifice and stigma' refers to costly and unproductive prescriptions of religious groups, susceptible of screening out potential free-riders. His model, however, does not address the question of why certain churches, generally older and larger, become more liberal and reduce their costly requirements. This paper acknowledges the critical role of "sacrifice and stigma" in the functioning of religious groups, and conceptualizes them as means of not only screening but also signaling the degree of commitment of members, resolving the prisoner's dilemma. I argue that as the share of church-members who are affiliated by birth increases, the association between the required "sacrifice and stigma" and the actual contribution of church-members weakens, reducing the accuracy of the signal. I model the setting as a differential game, derived from Benchekroun and Long (2008). I show that in such circumstances, a reduction in the required "sacrifice and stigma" increases the steady-state level of contribution to the church. The model explains why churches with near universal affiliation rates in their reference population tend to become more liberal overtime. The relevance of the model's predictions is discussed in relation to the evolution of the Roman Catholic Church of Québec, Canada.

Optimal control problem for two-sector economic model with Cobb–Douglas production function

Mikhail V. Orlov (1), Yury N. Kiselev (1), Sergey M. Orlov (1),

(1) Applied Mathematics Faculty, Lomonosov Moscow State University, Russia

A few resource allocation problems in a two-sector model with the Cobb–Douglas production function and different functional costs are considered. The planning horizon may be finite or infinite. A constructive description of the optimal solution is proposed in each case. The problems solution is based on the Pontryagin maximum principle. The optimal solution may have a singular arc. In all cases the correspondent singular arc is described by an analytical way. Moreover, the singular trajectory is the same in all problems. The optimality of an extreme solution is proved using the theorem on the sufficient optimality conditions in terms of constructions of the maximum principles. One of the studied problems with different production functions is open to

biological interpretation under the model of balanced growth of plants within a given limited time span.

Revisiting the Lucas model

Bernhard Skritek (1,2), Jesus Crespo Cuaresma (1,3,4,5), Klaus Prettner (2), Alexia Prskawetz (1,2,4), Elena Rovenskaya (1),

(1) Vienna University of Technology, Institute for Statistics and Mathematical Methods in Economics, Austria (2) International Institute of Applied Systems Analysis (IIASA) (3) Vienna University of Economics and Business, Austria (4) Wittgenstein Center for Demography and Global Human Capital (IIASA, VID/OEAW,WU) (Austrian Institute of Economic Research (WIFO))

We revisit the influential economic growth model by Lucas [1], assuming that households optimally allocate consumption and education over the life-cycle given exogenous interest rates and wages. We thus present a simplified version of the model which abstracts from general equilibrium effects on market prices. We show that in such a partial equilibrium setting, the two state optimization problem (with physical capital and human capital as the state variables) can be decomposed into two single-state optimal control models. This transformation allows us to rigorously prove the existence of a singular solution along a balanced growth path. If a singular solution exists for the infinite time horizon, infinitely many optimal controls for the individual household problem exist. Different methods to overcome the problem of solution selection are discussed. Furthermore, we give an outlook of a Lucas model with heterogeneous agents.

[1] Lucas, R. Jr. On the mechanics of economic development. *Journal of Monetary Economics*, 22(1):3–42.

8. Set-valued and variational analysis 1 — 14:00 – 15:40

***Chair*: R. Cibulka** — **Prechtlsaal I**

Metric regularity in optimal control and differential inclusions

Alexander D. Ioffe (1),

(1) Department of Mathematics, Technion, Haifa, Israel

I shall discuss some new and fairly simple proofs of certain well known results (Filippov's existence and relaxation theorems for differential inclusions, necessary condicitons for variational problems with state constraints) based on methods of the theory of metric regularity.

A nonsmooth Robinson's inverse function theorem in Banach spaces

Asen L. Dontchev (1,2),

(1) American Mathematical Society, Ann Arbor, USA (2) University of Michigan, Ann Arbor, USA

In a recent paper, Izmailov derived an extension of Robinson's implicit function theorem for nonsmooth generalized equations in finite dimensions, which reduces to Clarke's inverse function theorem when the generalized equation is just an equation. Páles gave earlier a generalization of Clarke's inverse function theorem to Banach spaces by employing Ioffe's strict pre-derivative. In this talk we present a generalization of both theorems of Izmailov and Páles to nonsmooth generalized equations in Banach spaces.

Second order tangents and normals and necessary optimality conditions in optimal control

Hélène Frankowska (1),

(1) CNRS and University Pierre et Marie Curie, Paris, France

We discuss notions of second order tangents and normals to sets and apply them to investigate second order optimality conditions for the Mayer problem in optimal control. We work with a general control set and first derive a necessary condition in the integral form that correspond to second order variations of trajectories of control system. Then we show how this integral condition leads to various pointwise conditions in quite general settings.

[1] H. Frankowska and N. P. Osmolovskii. Second-Order Necessary Optimality Conditions for the Mayer Problem Subject to a General Control Constraint. https://hal.inria.fr/hal-01088904/ 2014, submitted.

[2] H. Frankowska and D. Hoehener. Second Order Adjoint and Maximality Condition for the Mayer Problem, 2015. (submitted)

Open mapping theorems with constrained domains

Radek Cibulka (1), Marián Fabian (2), Alexander D. Ioffe (3),

(1) NTIS - New Technologies for the Information Society and Department of Mathematics, Faculty of Applied Sciences, University of West Bohemia, Univerzitní 22, 306 14 Pilsen, Czech Republic (2) Mathematical Institute of Czech Academy of Sciences, Žitná 25, 115 67 Praha 1, Czech Republic (3) Department of Mathematics, Technion, Haifa, Israel 32000

We survey some recent regularity statements in variational analysis. We focus on theorems guaranteeing regularity of a mapping in question provided that it is approximated by a convex bunch of linear surjective operators. We focus in particular on the technology of proofs of such statements. We start from Ioffe's regularity criterion that substitutes complicated iterative procedures. Then we discuss the question of necessity of using the deep Brouwer's fixed point theorem and Michael's theorem on continuous selections of lower semicontinuous functions. Further, we highlight the use of smooth renorming of the spaces in question, and finally we close the bow of arguments by a suitable separable reduction. All this enables to refrain from using Brouwer's and Michael's theorems. In addition, our results hold also in reference points which may not belong to the interior of the domain of the considered mapping. The lecture is based on a forthcoming paper by Radek Cibulka, Marián Fabian and Alexander Ioffe: On primal regularity estimates for single-valued mappings.

9. PDE-constrained optimization 1 — 14:00 – 15:40

***Chair*: I. Neitzel, A. Rösch** — **Prechtlsaal II**

Optimal control for fracture propagation modeled by a phase-field approach

Ira Neitzel (1), Thomas Wick (2), Winnifried Wollner (3),

(1) Lehrstuhl für Optimalsteuerung, Technische Universität München, Germany (2) RICAM, Austrian Academy of Sciences, Linz, Austria (3) Department of Mathematics, University of Hamburg, Germany

We are concerned with an optimal control problem governed by a fracture model using a phase-field technique. To avoid the non-differentiability due to the irreversibility constraint, the fracture model is relaxed using a penalization approach. Due to the removal of L^∞ bounds on the phase-field, well posedness of the penalized fracture model needs to be analyzed. Existence of a solution to the penalized fracture model is shown and utilized to establish existence of at least one solution for the regularized optimal control problem.

On the regularity of the solutions of Dirichlet optimal control problems in polygonal domains

Thomas Apel (1), Mariano Mateos (2), Johannes Pfefferer (1), Arnd Rösch (3),

(1) Universtät der Bundeswehr München, Germany (2) Universidad de Oviedo, Spain (3) Universität Duisburg–Essen, Germany

A linear quadratic Dirichlet control problem posed on a possibly non-convex polygonal domain is analyzed. Detailed regularity results are provided in classical Sobolev (Slobodetskiĭ) spaces. Whereas in many other control problems the regularity of the unconstrained solution is better than that of the constrained solution, we encounter here the phenomenon that the constraint inhibits poles of the unconstrained solution. For that reason the regularity of the constrained control is determined by the largest convex angle but the regularity of the unconstrained control is determined by the overall largest angle.

On the coupling of regularization and discretization for Bingham fluids

Juan Carlos De los Reyes (1), Winnifried Wollner (2),

(1) Centro de Modelización Matemática (MODEMAT), Escuela Politécnica Nacional Quito, Quito, Ecuador (2) Department of Mathematics, University of Hamburg, Hamburg, Germany

We investigate the coupling between regularization parameters and mesh size step for the numerical solution of Bingham fluids. A local Huber type regularization is considered in combination with a finite element discretization. We improve classical theoretical results concerning the convergence order of the discretization and propose an algorithm for the efficient solution of the problem. The results are experimentally verified.

Optimal control of a non-smooth Cahn-Hilliard-Navier-Stokes system

Michael Hintermüller (1), Tobias Keil (1), Donat Wegner (1),

(1) Department of Mathematics, Humboldt-Universität zu Berlin, Germany

The optimal boundary control of a time-discrete Cahn-Hilliard-Navier-Stokes system is studied. A general class of free energy potentials is considered which, in particular,includes the double-obstacle potential. The latter homogeneous free energy density yields an optimalcontrol problem for a family of coupled systems, which result from a time discretization ofa variational inequality of fourth order and the Navier-Stokes equation. The existence of an optimalsolution to the time-discrete control problem as well as an approximate version is established.The latter approximation is obtained by mollifying the Moreau-Yosida approximation of the doubleobstaclepotential. First order optimality conditions for the mollified problems are given, and inaddition to the convergence of optimal controls of the mollified problems to an optimal control ofthe original problem, first order optimality conditions for the original problem are derived througha limit process. The newly derived stationarity system is related to a function space version ofC-stationarity. The talk ends by a report on numerical tests.

10. Regime switching in differential games — 14:00 – 15:40

***Chair*: S. Wrzaczek** — **Festsaal**

Building a canal and transferring water between two river basins

Francisco Cabo (1), Katrin Erdlenbruch (2), Mabel Tidball (3),

(1) IMUVa, Universidad de Valladolid, Valladolid, Spain (2) Irstea, UMR G-EAU, Montpellier, France (3) INRA, UMR Lameta, F-34000 Montpellier, France

Following [1] we study a situation where precipitations are relatively abundant in one river basin while the productivity of water is relatively high in a neighboring river basin. Moreover we consider that the economic benefits linked with the transfer of water from the former to the latter overcome the construction costs of the required infrastructure. In this case, the two regions might have an incentive to cooperate and construct a canal. This problem is analyzed distinguishing two periods. Once the canal is under operation, the price and the size of the water transfer are determined from the dynamic interaction between the two river basins. A non-cooperative differential game is played between the donor who determines the supply of water and the recipient who chooses the demand for water and the investment in alternative water supplies. The construction of the canal requires a first period of cooperation and joint investment. The duration of this cooperative period is not fixed ex ante but dependent on the optimal investment decisions. In this cooperative period the costs must be shared taking into account each region's future gains once the canal is finished. Along the lines of [2], a side-payment is defined to guarantee overall individual rationality: each region is globally better off by agreeing to cooperate first and convey/receive water later on than in the no transfer scenario. Further, the side-payment must guarantee time consistency: individual rationality in every subgame along the cooperative trajectory.

[1] F. Cabo, K. Erdlenbruch and M. Tidball. Dynamic management of water transfer between two interconnected river basins. *Resource and Energy Economics*, 37, 17–38, 2014.

[2] S. Jørgensen and G. Zaccour. Time consistent side payments in a dynamic game of downstream pollution. *Journal of Economic Dynamics and Control*, 25, 1973–1987, 2001.

On a class of hybrid differential games

Dmitry Gromov (1), Ekaterina V. Gromova (1),

(1) Faculty of Applied Mathematics and Control Processes, St.-Petersburg State University, St.-Petersburg, Russia

We consider a class of bimodal differential games with switching which generalizes that presented in [1]. Both state-dependent and time-dependent switching formulations are addressed. A dynamic programming formulation of the associated optimal control problem(s) is presented along the lines of [2,3].

We discuss a number of game theoretic problems that arise within the hybrid context such as the calculation of Nash equilibria and the choice of the optimality principle in the cooperative form of the game.

The obtained results are illustrated by an example of a hybrid game of pollution control.

[1] D. Gromov, E. Gromova, Differential Games with Random Duration: A Hybrid Systems Formulation. *Contributions to game theory and management*, Vol. VII, 2014.

[2] A. Pakniyat, P.E. Caines. On the minimum principle and dynamic programming for hybrid systems. *Proceedings of the 19th International Federation of Automatic Control World Congress*, IFAC. 2014.

[3] P.E. Caines, M. Egerstedt, R. Malhame, A. Schoellig. A hybrid Bellman equation for bimodal systems. In: *Hybrid Systems: Computation and Control*. pp. 656-659. Springer, 2007.

Optimal management with hybrid dynamics

Puduru V. Reddy (1), J. M. Schumacher (2), Jacob Engwerda (2),

(1) GERAD, HEC Montréal, Canada (2) Department of Econometrics and Operations Research, Tilburg University, Tilburg, The Netherlands

Most of the optimal decision making problems studied in economics and ecology are complex in nature. These complexities generally arise while modeling the inherent behavior of the dynamic environment, which includes agents interacting with the system. Modeling with hybrid systems capture some of these complex situations. The behavior of such systems is described by the integration of continuous and discrete dynamics. An abrupt change in the discrete state of the system is called a switch. If a decision maker influences a switch then it is said to be controlled/external, whereas an internal switch generally results when the continuous state variable satisfies some constraints. Threshold effects are autonomous switches that happen when

the continuous state variable hits a boundary. Some examples in this direction are, a firm going bankrupt when its equity is negative and regime shifts in ecology. Optimal control of hybrid systems has received considerable interest in control engineering. These works include formulation of the necessary conditions and computational algorithms towards solving these conditions.

In this work, we introduce a class of discounted autonomous infinite horizon optimal control problems with endogenous switching. We review the necessary conditions for these class of problems. Further, for one dimensional state space, we provide some specific results to analyze the optimality conditions. Using these methods we analyze the shallow lake problem where the lake dynamics is modeled using hybrid dynamics.

Advertising strategies to counteract an incoming competitor

Stefan Wrzaczek (1), Alessandra Buratto (2), Valentina Mazzocco (2),

(1) Department of Business Administration, University of Vienna, Vienna, Austria (2) Department of Pure and Applied Mathematics, University of Padua, Padova, Italy

We analyse a market in which a new competitor is going to enter into the market. In particular, we study the different advertising policies that a monopolist firm can adopt, before and after the entrance of the rival.

Once formalized the problem in the context of the differential game theory, the following three scenarios are explored: In the first one we assume that the original firm is myopic and it does not expect that the competitor will arrive, in the second scenario the firm knows that the competitor will arrive at a given (and known) instant, finally, in the third scenario, the original firm only knows that the arrival will occur at a given constant rate, but the precise instant is not known. After solving the optimal control problem for the monopolist, we characterise the Feedback Nash Equilibrium of the game, in order to compare the different behaviours from a marketing point of view.

11. Resource economics — 14:00 – 15:40

***Chair*: Session Chair — Böcklsaal**

Optimal managed harvesting under imperfect fishery control

Domenico De Giovanni (1), Fabio Lamantia (1),

(1) Department of Economics Statistics and Finance, University of Calabria, Cosenza, Italy

We analyze the optimal harvesting rule of a monopolist in a managed single-specie fishery environment where the fishery control is assumed to be imperfect. The monopolist's control space consists of legal and illegal actions. Illegal actions might be detected at random times, in which case the monopolist is subject to a deterrence scheme in line with the Common Fishery Policy implemented by the European Union. We first model the monopolist's decision problem as a piece-wise deterministic optimal control problem. Then we use a semi-Lagrangian discretization scheme to derive numerical value functions and policy rules.

We show that the introduction of the management policy, together with the inability of the regulator to perfectly monitor fishing activities, creates an incentive to over-harvest, not only beyond the allowed quota, but also beyond the harvest in a unregulated but otherwise equal situation. In terms of the dynamics of the controlled process, this incentive causes the creation of an attractive sliding motion, which makes the resource level oscillate around the point of discontinuity of the harvesting rule. The possible convergence to this long–run level rises the question on how the regulator may modify the quota and the effort in order to increase the long–run equilibrium of the resource. We find that the over-harvesting effect is particularly pronounced at lower levels of the legal quota. The main implication is a shift toward right of the discontinuity, and a lower value of the long-run equilibrium. Also, lower quotas require more regulatory control effort to avoid the convergence of the resource toward a low level. We finally show that reinforcing the deterrence scheme accompanied with a sufficiently high level of the legal quota completely eliminates this effect.

Sharing a groundwater resource in a context of regime shifts

Julia de Frutos Cachorro (1), Katrin Erdlenbruch (2), Mabel Tidball (1),

(1) INRA, UMR 1135 LAMETA, F-34000 Montpellier, France (2) IRSTEA, UMR G-EAU, F-34196 Montpellier, France

We consider the exploitation of a common groundwater resource for irrigation as a differential game, that is, by taking into account the strategic and dynamic interactions between the users of the resource. In particular, we use the Rubio and Casino adaptation (see [2]) of the Gisser and Sánchez model (see [1]), where we introduce a sudden change in the dynamics of the resource, namely a decrease in the recharge rate of the aquifer. Such a shock (also called regime shift) may occur due to a decrease in mean precipitation that leads to a decrease in the recharge of the aquifer, or it may correspond to the abstraction of a certain amount of water that is dedicated to other uses in the case of a drought, such as filling drinking water reservoirs. We first compare the effects of the regime shift on the exploitation of the aquifer according to different information structure available to the users of the resource: open loop and feedback strategies. Furthermore, we propose an alternative and more realistic information structure to the open-loop solution which we call ”piecewise open-loop strategy”. We then compute the socially optimal case to estimate (in terms of stock) the inefficiency of private solutions. We show analytically that different solutions (at the steady state) do not depend on the intensity of the shock, but on the value of the recharge rate upon occurrence of the shock. Moreover, we show that solutions get closer at the steady state for lower values of recharge rates. We finally apply the game to the particular case of the Western La Mancha aquifer, with the aim of estimating (in terms of welfare) the inefficiency of private exploitation with regards to the characteristics of the shock. We show that the loss of welfare due to private exploitation is maximal for low-intense or later shocks. Thus, a regulation of the aquifer through a centralized management is even more justified in a context of regime shifts, providing efficiency gains which can reach 40 millions of Euros.

[1] M. Gisser, D.A. Sánchez. Competition versus optimal control in groundwater pumping. *Water Resources Research*, 31, 638–642, 1980.

[2] S.J. Rubio, B. Casino. Competitive versus efficient extraction of a common property resource: The groundwater case. *Journal of Economics Dynamics and Control*, 25, 1117–1137, 2001.

Dynamic resource management under weak property rights: a tale of thieves and trespassers

Mauricio A. Rodriguez (1), Sjak Smulders (1),

(1) Department of Economics and Tilburg Sustainability Center, Tilburg University, Tilburg, The Netherlands

Using a differential game framework we study the management of an exhaustible natural resource when property rights are generally weak. We define generally weak property rights as those under which both the stock of the resource and the revenues from exploiting it are imperfectly protected, due to trespassing and theft respectively. From the legitimate owner's perspective, trespassing and theft have a fundamental effect on the inter-temporal trade-off governing the extraction decision: extracting the resource today protects it against trespassing but exposes it to theft. Our main results indicate that total depletion of the resource is always decreasing in the intensity of theft, and that when the trespassers are affected by theft there is over (under) extraction in equilibrium if theft intensity is low (high).

Accelerating the diffusion of a new product through marketing-operations interactions

Fouad El Ouardighi (1), Gustav Feichtinger (2), Gila Fruchter (3),

(1) ESSEC Business School, Paris, France (2) ORCOS, Vienna University of Technology, Vienna, Austria (3) Graduate School of Business, Bar-Ilan University, Israel

In this paper, we suggest a new model to account for mixed word of mouth and accelerate diffusion by involving operation management instruments in addition to marketing instruments. Operation management instruments are involved by shaping the product's design quality to influence the innovation effect, and the conformance quality to affect the imitation effect. A comparison of the model that uses marketing instruments (price and advertising) against the model that in addition uses operation management instruments (design quality and conformance quality) leads to interesting insights and conclusions.

12. Variational inequalities and applications in economics 1 — 16:00 – 17:15

***Chair*: M. Donato, M. Milasi,** — **Prechtlsaal I**

Dynamics and control of economic equilibrium

Terry Rockafellar (1),

(1) University of Washington, Seattle, USA

Economic equilibrium, in its most basic model, concerns the existence of market prices that balance supply with demand when agents optimize utility in trading their given holdings of goods for better holdings under their budget constraints. Economists have come to think that equilibrium in this sense is a fragile concept subject to extreme instability even under seemingly strong assumptions on utility functions which in particular require all agents to maintain positive holdings in all goods. Recently, though, it has been shown that all this has been based on a misperception. Under quite reasonable assumptions, allowing also for some goods to be held by an agent in null amounts, if an economy is at an equilibrium in its prices and holdings, and a small change is made to those holdings, there will be a unique way to adjust to a new equilibrium nearby.

The especially interesting consequence of this fact is that it leads, through passage to infinitesimal changes in holdings, to a sort of differential equation describing how prices and holdings should evolve in time in response a external influences. Those influences could be in the form of dynamical control by an external agent. This mathematical development is unprecedented in economics but also mathematically not fully complete for reasons that are interesting in themselves.

Optimality conditions for constrained vector equilibrium problems and applications

Maria B. Donato (1),

(1) Department of Mathematics and Computer Science, University of Messina, Messina, Italy

The purpose of this talk is to establish necessary and sufficient conditions for the solution of a vector equilibrium problem with both cone and equality constraints among infinite dimensional spaces. The Karush-Kuhn-Tucker conditions for the solution to the vector equilibrium problem are derived, without requiring that the ordering cone which defines the cone constraints has nonempty interior. The main result is obtained by introducing a new constraint qualification condition which use the tangent cone concept to the image space.

On the Pontryagin maximum principle in Banach spaces

Mikhail I. Krastanov (1, 2), Nadezhda K. Ribarska (1, 2),

(1) Faculty of Mathematics and Informatics, University of Sofia, James Bourchier Boul. 5, 1164 Sofia, Bulgaria (2) Institute of Mathematics and Informatics, Bulgarian Academy of Sciences, Acad. G. Bonchev str., block 8, 1113 Sofia, Bulgaria

A basic idea of the classical approach for obtaining necessary optimality conditions in optimal control is to construct suitable “control variations”. We use it to prove a Pontryagin maximum principle for infinite-dimensional optimal control problems in arbitrary Banach state space. The proof is based on an abstract result on a non-separation property of two closed sets.

13. Set-valued numerics and control 2 — 16:00 – 17:15

***Chair*: R. Baier, E. Farkhi** — **PrechtlsaalII**

On the solvability of relaxed one-sided Lipschitz inclusions in Hilbert spaces

Janosch Rieger (1), Tobias Weth (1),

(1) Institute of Mathematics, Goethe-University Frankfurt, Frankfurt am Main, Germany

The relaxed one-sided Lipschitz property was first identified as a stability criterion for time-dependent differential inclusions. Later, surjectivity of ROSL mappings with negative ROSL constant and therefore solvability of the corresponding algebraic inclusions was shown, but no information on the localization of the solutions was obtained.

In this talk, I will present a generalization of the solvability result from [1] to infinite-dimensional Hilbert spaces and Gelfand triples. It reveals certain aspects of the behavior of the inverses ROSL mappings and gives rise to a numerical algorithm for the solution of ROSL algebraic inclusions, which is applied to a system of elliptic differential inclusions.

[1] W.-J. Beyn and J. Rieger. An iterative method for solving relaxed one-sided Lipschitz algebraic inclusions. *Journal of Optimization Theory and Applications*, 164(1):154–172, 2015.

Various notions of one-sided Lipschitz continuity based on set differences

Robert Baier (1), Elza Farkhi (2),

(1) Chair of Applied Mathematics, University of Bayreuth, Germany (2) School of Mathematical Sciences, Tel-Aviv University, Israel

Lipschitz and the weaker *One-sided Lipschitz (OSL)* regularity of set-valued maps are applied e.g. for the analysis of approximations of reachable sets of nonlinear control problems. Although OSL maps may be discontinuous, general convergence order $1/2$ (up to 1 in special cases) with respect to the time step-size can be obtained for the Euler method applied to OSL systems.

In the talk we present a unified framework to obtain various notions of Lipschitz and one-sided Lipschitz regularity for set-valued maps, based on set differences. The set differences under discussion – the algebraic, geometric, metric and Demyanov difference – yield known or new Lipschitz and OSL notions. Lipschitz maps with respect to the Hausdorff or the Demyanov metric, as well as strengthened OSL maps are discussed as examples.

This framework allows to establish the hierarchy of different Lipschitz or OSL notions. Furthermore, D-Lipschitz and D-OSL maps with convex images are characterized by the existence of generalized Steiner selections being uniformly Lipschitz resp. OSL. For univariate maps with compact images similar results are derived by using metric selections.

Space discretization robust viability kernels, and their application to autonomous driving

Alexander Liniger (1), John Lygeros (1),

(1) Automatic Control Lab, ETH Zürich, Zürich, Switzerland

The discrete time viability kernel of a set-valued map is typically approximated by discretizing space and making use of the viability kernel algorithm in the finite space, as proposed in [1]. The approximation is guaranteed to converge as the discretization goes to zero and the resulting finite viability kernel is a subset of the discrete viability kernel of an inflated set-valued map. However, the inflation process can lead to errors: for a viable grid point, there may exist points within a box for which no viable solution exists. This can lead to problems when reconstructing a viable feedback law.

We propose to characterize the uncertainty coming from the space discretization as an additive disturbance term in the system dynamics and formulate the viability computation problem as a dynamic game between this uncertainty and the controls. The victory domain of the controls can be calculated using the discriminating kernel algorithm [2]. We show that this discriminating kernel approximation is also an inner approximation and converges to the viability kernel as the discretization gets finer. Moreover, all points in a discretization box around an element of the kernel are themselves viable. The new algorithm is demonstrated on two numerical examples, a benchmark linear system and an autonomous driving testbed.

[1] P. Saint-Pierre. Approximation of the Viability Kernel. *Applied Mathematics and Optimization*, 29:187–209, 1994.

[2] P. Cardaliaguet, M. Quincampoix, P. Saint-Pierre. Set-Valued Numerical Analysis for Optimal Control and Differential Games. *Stochastic and differential games*. Springer, 1999.

14. Health care 1 — 16:00 – 17:15

***Chair*: M. Kuhn, S. Wrzaczek** — **Festsaal**

The economics of health demand and human aging: health capital vs. health deficits

Holger Strulik (1), Carl-Johan Dalgaard (2),

(1) University of Goettingen, Department of Economics, Platz der Goettinger Sieben 3, 37073 Goettingen, Germany (2) Department of Economics, University of Copenhagen, Oester Farimagsgade 5, building 26, DK-1353 Copenhagen, Denmark

This paper provides a rigorous comparative analysis of assumptions and predictions of alternative economic theories of health demand and human aging. The health-capital model, based on Grossman (1972) and the health-deficit model, based on Dalgaard and Strulik (2014). We show that both theories lead to fundamentally different predictions of health behavior and human life histories. We find that the health-deficit model provides a consistent approach to health behavior and aging whereas the health-capital model generates predictions that are hard to square with the stylized facts. We argue that the root of the disagreement of the theories is the following: The health-capital model postulates that of two people of the same age the healthier one looses more health in the next instant whereas the health-deficit model, based on insights from modern gerontology, proceeds from the opposite assumption.

Longevity shocks with age-dependent productivity growth

Ben J. Heijdra (1), Laurie S.M. Reijnders (1),

(1) University of Groningen, Groningen, The Netherlands

The aim of this paper is to study the long-run effects of a longevity increase on individual decisions about education and retirement, taking macroeconomic repercussions through endogenous factor prices and the pension system into account. We build a model of a closed economy inhabited by overlapping generations of finitely-lived individuals whose labour productivity depends on their age through the build-up of labour market experience and the depreciation of human capital. We make two contributions to the literature on the macroeconomics of population ageing. First we show that it is important to recognize that a longer life need not imply a more productive life and that this matters for the affordability of an unfunded pension system. Second, we find that factor prices could move in a direction opposite to the one accepted as conventional wisdom following an increase in longevity, depending on the corresponding change in the age-productivity profile.

On the role of a health care in general equilibrium

Michael Kuhn (1), Ivan Frankovic (1), Stefan Wrzaczek (2),

(1) Wittgenstein Centre (OeAW/VID, IIASA, WU), OeAW/VID, Austria (2) University of Vienna, Department of Business Administration, Austria

We study the role a health care market plays within a continuous time economy of overlapping generations subject to endogenous mortality. The economy consists of two sectors: final goods production and a health care sector, selling medical services to individuals. Individuals demand health care with a view to lowering mortality and morbidity over their life-cycle. We derive the age-specific individual demand for health care based on the value of life as well as the resulting aggregate demand for health care across the population. We then characterize the general equilibrium allocation of this economy, providing both an analytical and a numerical representation.

We study the allocational impact of population change (an exogenous baby boom/bust); exogenous medical progress; technological progress in final goods production; and health policy design (e.g. a tax/subsidy on health care). We place particular emphasis (i) on understanding how the value of life (and, thus, the demand for health care) responds to economic growth, medical or policy change in general equilibrium rather than partial equilibrium; and (ii) on studying the efficiency of the decentralized allocation by contrasting it against the central planner solution.

15. Pollution abatement in dynamic oligopoly games 16:00 – 17:15

***Chair*: L. Lambertini** **Böcklsaal**

Emission taxation, green innovations and inverted-U aggregate R&D efforts in a linear state oligopoly game

Davide Dragone (1), Luca Lambertini (1), Arsen Palestini (2),

(1) Department of Economics, University of Bologna, Bologna, Italy (2) MEMOTEF, Sapienza University of Rome, Rome, Italy

We revisit the well known differential Cournot game with polluting emissions dating back to Benchekroun and Long (1998), proposing a version of the model in which environmental taxation is levied on emissions rather than the environmental damage. This allows to attain strong time consistency under open-loop information, and yields two main results which can be summarized as follows: (i) to attain a fully green technology in steady state, the regulator may equivalently adopt an appropriate tax rate (for any given number of firms) or regulate market access (for any given tax rate); (ii) if the environmental damage depends on emissions only (i.e., not on industry output) then the aggregate green R&D effort takes an inverted U shape, in accordance with Aghion *et al.* (QJE, 2005).

Pollution accumulation and abatement policies under bilateral duopolistic competition

Fouad El Ouardighi (1), Jeong Eun Sim (2), Bowon Kim (2),

(1) ESSEC Business School, Cergy Pontoise, France (2) KAIST Business School, Seoul, Korea

The existing literature in environmental economics evaluates the improper internalization of pollution externalities in a horizontal setting (i.e., two producers, nations, etc), but disregards the influence of both vertical and horizontal competition. According to the existing literature, strategy types (Basar and Olsder, 1998; Dockner et al., 2000) do matter in a non-competitive, horizontal setting where producers draw utility from polluting emissions as a by-product of production and incur a cost related to the pollution stock, that is, open-loop Nash equilibrium strategies result in a lower steady state pollution stock than feedback Nash equilibrium strategies. This research analyzes the impact of both market structure and strategy types on the evolution of pollution accumulation in a bilateral duopoly setup.

On the optimal number of firms in a polluting oligopoly: a differential game with open-loop and linear feedback solutions

Luca Lambertini (1), Andrea Mantovani (1),

(1) Department of Economics, University of Bologna, Bologna, Italy

We revisit the dynamic Cournot game with polluting emissions from the standpoint of the well established discussion about the optimal number of firms in the commons. Leaving aside

any regulation of firms' behavior, we show that under linear feedback rules the industry structure which maximizes social welfare also minimizes the volume of emissions. This clearly does not apply under open-loop information.

16. Variational inequalities and applications in economics 2 — 17:30 – 18:45

***Chair*: M. Donato, M. Milasi** — **PrechtlsaalI**

Evolutionary variational inequality formulation of generalized Nash equilibrium problem with application to electricity markets

Didier Aussel (1), Rachana Gupta (2), Aparna Mehra (2),

(1) Lab. PROMES, University of Perpignan Via Domita, Perpignan, France (2) Department of Mathematics, Indian Institute of Technology - Delhi, New Delhi-16, India

For time dependent generalized Nash equilibrium problem a reformulation in evolutionary variational inequality is proved in the general setting of quasiconvex decision functions. An existence result for time dependent generalized Nash equilibrium problem is deduced and application to dynamic electricity market is also considered.

Semistrictly quasiconvex approximation and applications to variational problem

Monica Milasi (1),

(1) Department of Mathematics and Computer Science, University of Messina, Messina, Italia

In this talk we show how to approximate, in the sense of continuous convergence, a quasiconvex function with a sequence of semistrictly quasiconvex functions. Next, the former results are applied in order to get some general theorems of existence of solution of an equilibrium problem. In particular we reformulate an economic equilibrium problem in terms of a suitable variational inequality. Hence, we use our approximation in order to give the existence of equilibrium when utility functions are quasiconcave and continuous.

Variational approach for a general financial equilibrium problem

Sofia Giuffrè (1), Patrizia Daniele (2),

(1) DIIES, Mediterranea University of Reggio Calabria, Italy (2) Department of Mathematics and Computer Science, University of Catania, Italy

This talk is concerned with a general model of financial flows and prices, evolving in time, which are related to individual entities, called sectors, that invest in financial instruments as assets and as liabilities.

In [1] the authors give for this model the equilibrium conditions in a dynamic sense, express them in terms of an evolutionary Variational Inequality, provide existence theorems and present

a dual formulation of the financial equilibrium, in which the Lagrange variables $\rho_j^{*1}(t)$ and $\rho_j^{*2}(t)$, called "deficit" and "surplus" variables, appear. These variables play a fundamental role in order to analyze the model and the possible insolvencies and to achieve suggestions for the management of the world economy (see the Deficit Formula, the Balance Law and the Liability Formula). Aim of the talk is to present the model in detail, together with the equilibrium conditions and their variational inequality formulation, present the dual formulation and prove the continuity of the Lagrange functions "deficit" and "surplus". The regularity result allows to apply a calculus procedure in order to compute the solutions.

[1] A. Barbagallo, P. Daniele, S. Giuffrè and A. Maugeri, Variational approach for a general financial equilibrium problem: The Deficit Formula, the Balance Law and the Liability Formula. A path to the economy recovery. *European Journal of Operation Research*, 237: 231-244, 2014.

[2] P. Daniele, S. Giuffrè, Functional Inequalities and Regularity of the Deficit and Surplus Variables in the Financial Equilibrium Problem, submitted

17. Set-valued numerics and control 3 — 17:30 – 18:45

***Chair*: R. Baier, E. Farkhi** — **Prechtlsaal II**

Optimization-based subdivision algorithm for reachable sets

Wolfgang Riedl (1),

(1) Department of Mathematics, University of Bayreuth, Bayreuth, Germany

The reachable set (sometimes also called attainable set) at a given time T of a nonlinear control system is the set of the endpoints of all feasible solutions. In this talk it is shown how an optimization-based algorithm to calculate discrete reachable sets can be improved using a subdivision technique studied e.g. by M. Dellnitz and A. Hohmann, L. Grüne. The basic optimization-based approach discretizes the state-space into equidistant grid points and an optimization problem has to be solved for each of these. Using subdivision an adaptive version of this algorithm can be formulated to improve the performance by reducing the number of grid points located far away from the reachable set.

The talk illustrates how an OCP-solver based on direct discretization can be applied to calculate discrete reachable sets and shows some strategies to increase the stability of the OCP-solver for this class of problems. The subdivision approach will be introduced and numerical examples will be shown to demonstrate the improvements of this adaptive algorithm.

Computation and visualization of control systems reachable sets with parallel processing on Graphics Processing Units

Alina O. Novikova (1),

(1) Lomonosov Moscow State University, Russia

The solution of a wide range of optimal control problems involves constructing reachable sets and their analogs. The pixel method of constructive description and determination of the reachable sets for a given controlled system is described and implemented. For the initial set we build the characteristic matrix in the following way: all the set is covered up with the grid. The size of this grid depends on our implementation. If the element from the initial set reaches the grid node, the corresponding position of the characteristic matrix is 1, otherwise it is 0. Then at every step we work with characteristic matrix. We go from one moment of time to another with time steps and we solve the Cauchy problem every time.

In the pixel method parallel computing is applied to characteristic matrix work, because the work with every pixel element goes independently in parallel way. Presented algorythm is used to solve the problem of construction of reachable sets numerically. CUDA (Compute Unified Device Architecture) technology allows to use Graphics Processing for parallel processing Units. CUDA is a complex of software and hardware tools. CUDA is a parallel computing platform and a programming model. It dramatically increases the computing performance by increasing the power of the graphics processing units (GPU).

The effectiveness of the approach is demonstrated on several examples. Based on realistic numerical values for control examples we conclude that the computed reachable sets are in fact full thrust. The approach may be applied to multidimensional problems.

Optimal population structures in heterogeneous epidemiological models

Tsvetomir Tsachev (1), Vladimir M. Veliov (2), Andreas Widder (2),

(1) Institute of Mathematics and Informatics, Bulgarian Academy of Sciences, Sofia, Bulgaria (2) ORCOS, Vienna University of Technology, Vienna, Austria

We consider the spreading of an infectious disease in a heterogeneous population. The heterogeneity is realised by assigning each individual in the population a parameter out of a given parameter space. The initial distribution can have a strong influence on the progress of the disease. However, this distribution is generally not exactly known; only some aggregate and/or set-membership information is available. The trajectories of the disease which are consistent with this information form a tube, the time sections of which provide guaranteed set-membership estimations of the state of the disease. We develop a numerical procedure for determining such estimations and apply it to study the guaranteed effect of various prevention policies.

18. Economic growth and demographic change 1 — 17:30 – 18:45

***Chair*: K. Prettner, A. Prskawetz** — **Festsaal**

Population and economic growth under different growth engines

Alberto Bucci (1), Xavier Raurich (2),

(1) University of Milan, Milan, Italy (2) Universidad de Barcelona, Barcelona, Spain

Using a growth model with physical and human capital accumulation and R&D activity, this paper proposes an additional channel through which an increase in the population growth rate

may yield a non-uniform (i.e., a positive, or a negative, or else a neutral) impact on the growth rate of per-capita GDP, as available empirical evidence seems mostly to suggest. The proposed mechanism relies on the nature of the process of economic growth (whether it is fully or semi-endogenous), and the peculiar engine(s) driving economic growth (human capital investment, R&D activity, or both). The model can also explain the so-called "population-productivity reversal", that is the reduction in the growth rate of per capita GDP due to an increase in the population growth rate when R&D is an engine of economic growth.

Public education and R&D-based economic growth

Katharina Werner (1), Klaus Prettner (2),

(1) University of Goettingen, Goettingen, Germany (2) Vienna University of Technology, Vienna, Austria

We analyze the short- and long-run effects of public education on economic growth and welfare. In so doing, we extend an R&D-based economic growth model by including a governmental sector that levies labor income taxes and uses the proceeds to finance teachers. An increase in the tax rate reduces consumption possibilities, and the number of workers available for final goods production and research but at the same time it increases the educational resources per pupil. Consequently, economic growth slows down immediately but speeds up during the transition toward the long-run balanced growth rate. Due to the reduction on consumption, individual utility falls initially. This implies a dynamic tradeoff in the sense that current cohorts loose due to educational reform, whereas future cohorts gain. We show that there exists an interior welfare-maximizing level of the provision of public education and show that it is higher than the levels we typically observe in industrialized countries. Since the transitional effects of an education reform on growth and welfare can be negative, our framework has the potential to explain resistance against long-run welfare improving education reforms.

Child mortality and the growth drag of pollution

Andreas Schaefer (1),

(1) Swiss Federal Insitute of Technology (ETH), Center of Economic Research at ETH Zurich (CER-ETH), Zurich, Switzerland

In this paper we analyze the link between fertility, children's education and degradation of the environment through economic development as follows: Economic development may be conducive for children's survival probabilities, but may also generate via pollution adverse impacts on children's probability to survive to adulthood. Higher child mortality reduces the willingness of parents to invest in children's education and increases their desired level of fertility. In this context, economic inequality is not only decisive for human capital investments and the emergence of differential fertility between rich and poor households (de la Croix and Doepke, 2003), but also for agents' exposure to environmental pollution because wealthier households live in cleaner areas. The exposure to pollutants triggers again children's probability to survive to adulthood and the willingness of parents to invest in education. This is the key mechanism and the novelty of

our approach through which environmental conditions may impose a growth drag on the economy. Moreover, we argue that the adverse effect of inequality and pollution on economic growth is amplified, if the population group that is least affected decides about the level of tax-financed abatement measures,

19. Dynamic games in economics and management — 17:30 – 18:45

***Chair*: S. Chair** — **Böcklsaal**

Imitative dynamics for games with continuous strategy space

Man-Wah Cheung (1),

(1) Department of Economics, University of Wisconsin–Madison, USA

This paper studies imitative dynamics for population games with continuous strategy space. We define imitative dynamics—which include the replicator dynamic as a special case—as evolutionary dynamics that satisfy the imitative property and payoff monotonicity. Our definition of payoff monotonicity is different from the one defined in Oechssler and Riedel (2002). We find that our definition is better in capturing the payoff monotonicity (cf. Weibull (1995)) in finite strategy case in the literature, and Oechssler and Riedel (2002)'s definition is closer to aggregate monotonicity in the sense of Samuelson and Zhang (1992). We provide sufficient conditions for imitative dynamics and general evolutionary dynamics to be well-defined. Finally, we extend the general properties of imitative dynamics as well as global convergence and local stability results in potential games from finite strategy settings to continuous strategy settings.

Properties of feedback Nash equilibria in scalar LQ differential games

Jacob Engwerda (1),

(1) Dept. of Econometrics and O.R., Tilburg University, Tilburg, The Netherlands

In this presentation we consider linear feedback Nash equilibria of the scalar linear quadratic N-player differential game. We present a complete characterization of the solution structure of this game using a geometric approach. Furthermore we investigate the effect on this solution structure of some characteristics of the game, i.e.: the number of players; the entrance of new players; and the level of asymmetry. For that purpose we distinguish three types of the game: the economic game; the regulator game and the mixed game. The analysis is restricted to the case the involved cost depend only on the output and control variables.

On time preferences and bargaining

Sebastian Kodritsch (1,2),

(1) WZB Berlin Social Science Center, Germany (2) School of Business and Economics, Humboldt-University Berlin, Germany

This paper analyzes dynamically inconsistent time preferences in the seminal Rubinstein (1982) model of bilateral alternating-offers bargaining. I consider any continuous time preferences which satisfy a weak impatience property and study multiple-selves equilibrium for sophisticated players. I establish the sufficiency of simple penal codes akin to Abreu (1988) and characterize (i) the set of equilibrium outcomes for any given preference profile and (ii) the set of preference profiles for which equilibrium is unique. When both players are present-biased—a robust finding on the domain of primary rewards (e.g. any hyperbolic or quasi-hyperbolic discounting)—they are predicted to bargain efficiently: equilibrium is unique and stationary, agreement is reached immediately. By contrast, when some player finds a near-future period of delay relatively more costly than the first one from the immediate present—as suggested by recent experimental work for the domain of monetary rewards—multiple and non-stationary equilibria arise: in particular, the set of equilibrium outcomes then includes delayed agreement with gradually increasing offers.

Thursday, May 14

20. Plenary **08:45 – 09:35**

***Chair*: Session Chair** **Festsaal**

Partners or rivals? Strategies for the iterated Prisoner's Dilemma

Karl Sigmund (1), Christian Hilbe (2), Arne Traulsen (3),

(1) University of Vienna, Vienna, Austria (2) Harvard University, Cambridge, USA (3) Max-Planck-Institute for Evolutionary Biology , Plön, Germany

Within the class of memory-one strategies for the iterated Prisoner's Dilemma, we characterize partner strategies, competitive strategies and zero-determinant strategies. If a player uses a partner strategy, both players can fairly share the Pareto optimum; but a co-player preferring an unfair solution will be penalized by obtaining a reduced payoff. A player using a competitive strategy never obtains less than the co-player. A player using a zero-determinant strategy unilaterally enforces a linear relation between the two players' payoffs. These properties hold for every strategy used by the co-player, whether memory-one or not.

21. Set-valued and variational analysis 2 **10:00 – 11:15**

***Chair*: R. Cibulka** **PrechtIsaal I**

Jointly convex generalized Nash equilibria with an application to elliptic multiobjective optimal control

Axel Dreves (1), Joachim Gwinner (1),

(1) Department of Aerospace Engineering, Universität der Bundeswehr München, Neubiberg, Germany

We consider jointly convex generalized Nash equilibrium problems (GNEPs) in infinite dimensional spaces. We develop a discrete approximation procedure for the problem in a general Banach space, using a sequence of variational inequalities. Furthermore we provide, as it was done earlier in finite dimensional GNEPs, an optimization reformulation, which is exploited in order to design an algorithm that is convergent in Hilbert spaces. Then we reformulate a class of multiobjective optimal control problems, governed by elliptic partial differential equations, with control and state constraints, as a GNEP that can be shown to satisfy the assumptions of our theoretical considerations. Finally, we present some promising preliminary numerical results.

Composition set-valued mappings: metric subregularity and fixed points

Marius Durea (1), Radu Strugariu (1),

(1) Faculty of Mathematics, Al.I. Cuza University, Iaşi, Romania

We underline the importance of the parametric subregularity property of set-valued mappings, defined with respect to fixed sets. We show that this property appears naturally for some very simple mappings which play an important role in the theory of metric regularity. We show a result concerning the preservation of metric subregularity at generalized compositions. Then we obtain, on purely metric setting, several fixed point assertions for set-valued mappings in local and global frameworks

Optimality conditions for optimization problems with cardinality constraints

Michal Červinka (1,2), Christian Kanzow (3), Alexandra Schwartz (4),

(1) Institute of Information Theory and Automation, Czech Academy of Sciences, Czech Republic (2) Faculty of Social Sciences, Charles University in Prague, Czech Republic (3) University of Würzburg, Germany (4) Technical University of Darmstadt, Germany

Based on a recently introduced reformulation of optimization problems with cardinality constraints as a nonlinear program with continuous variables, we first define some problem-tailored constraint qualifications and then show how these constraint qualifications can be used to obtain suitable optimality conditions for cardinality constrained problems. In particular, we concern the concepts of S- and M-stationarity. Here, the (KKT-like) optimality conditions hold under much weaker assumptions than the corresponding result that is known for the somewhat related class of mathematical programs with complementarity constraints.

22. Stochastic control and applications 1 — 10:00 – 11:15

***Chair*: M. Falcone, H. Zidani** — **Prechtlsaal II**

State-constrained stochastic optimal control problems via reachability approach

Athena Picarelli (1), Olivier Bokanowski (2), Hasnaa Zidani (3),

(1) Mathematics Department, University of Oxford, Oxford, UK (2) Laboratoire Jacques-Louis Lions, Université Paris-Diderot (Paris 7), Paris, France (3) Mathematics Department (UMA), ENSTA ParisTech, Paris, France

This work is concerned with a general class of stochastic optimal control problems in presence of state-constraints. When state-constraints are taken into account and in absence of quite restrictive controllability assumptions on the dynamics, the continuity of the value function cannot be guaranteed and some well-known problems arise in its characterization as a viscosity solution of a Hamilton-Jacobi-Bellman equation. The approach proposed in this work leads to a characterization of the epigraph of the value function translating, at a first stage, the optimal control problem into a state-constrained stochastic target problem with unbounded controls. This new formulation of the problem has the advantage to allow to solve it by a level set approach, where the state-constraints can be managed by an exact penalization technique.

Stochastic target games and dynamic programming via regularized viscosity solutions

Bruno Bouchard (1, 2), Marcel Nutz (3),

(1) CEREMADE, Université Paris Dauphine, Paris, France (2) CREST, ENSAE-ParisTech, Malakoff, France (3) Columbia University, New-York, USA

We study a class of stochastic target games where one player tries to find a strategy such that the state process almost-surely reaches a given target, no matter which action is chosen by the opponent. Our main result is a geometric dynamic programming principle which allows us to characterize the value function as the viscosity solution of a non-linear partial differential equation. Because abstract measurable selection arguments cannot be used in this context, the main obstacle is the construction of measurable almost-optimal strategies. We propose a novel approach where smooth supersolutions are used to define almost-optimal strategies of Markovian type, similarly as in verification arguments for classical solutions of Hamilton-Jacobi-Bellman equations. The smooth supersolutions are constructed by an extension of Krylov's method of shaken coefficients. We apply our results to a problem of option pricing under model uncertainty with different interest rates for borrowing and lending.

Domain decomposition techniques for deterministic and stochastic optimal control problems

Simone Cacace (1), Maurizio Falcone (1),

(1) SAPIENZA Università di Roma, Rome, Italy

We present some recent developments in the numerical solution of Hamilton-Jacobi equations coming from deterministic and stochastic optimal control problems. We review the semi-Lagrangian discretization for first order equations, and we present the main features of the patchy domain decomposition method, an efficient and transmission-free parallel algorithm for deterministic control problems. Then, we extend these techniques to second order equations, namely to

nonlinear problems with possibly degenerate diffusion. To this end, we introduce suitable modifications of the semi-Lagrangian local solver, by combining some key concepts: self-dependency removal, numerical causality and upwind diffusion ball condition. Finally, we present several numerical tests to show the effectiveness of the proposed method.

23. Economic growth and demographic change 2 — 10:00 – 11:15

***Chair*: K. Prettner, A. Prskawetz** — **Festsaal**

Age-specific effects of mortality shocks and economic development

Shanka Chakraborty (1), Fidel Perez-Sebastian (2,3),

(1) University of Oregon, U.S.A. (2) University of Alicante, Spain (3) University of Hull, U.K.

We study the effects on income of mortality shocks that affect age groups differently. We find that when children are the most affected age-group income per capita increases; whereas when adults are the ones mainly affected, the impact on per capita income become negative.

Access to childcare and second child arrival in European countries

Hippolyte d'Albis (1), Paula Gobbi (2), Angela Greulich (3),

(1) Paris School of Economics - University Paris 1, Paris, France (2) Université catholique de Louvain, Louvain-la-Neuve, Belgium (3) Université Paris 1 Panthéon-Sorbonne, Paris, France

This paper shows that differences in fertility across European countries mainly emerge in the transition from the first to the second child and that childcare services enabling women to work are an important determinant for this transition to occur. The theoretical framework proposed accounts for these two findings: in countries where childcare coverage is low, there is a U-shaped relationship between a couples probability to have a second child and female potential wage, while in countries with easy access to childcare, this probability is positively related with womans potential wage. Both of these implications are confirmed empirically when mobilizing the European Survey of Income and Living Conditions (EU-SILC) for estimating womens probability of second birth as a function of education.

Education trap? Differential effect of the pension system on retirement and years of education by life expectancy

Miguel Sanchez-Romero (1,2), Alexia Prskawetz (1,2,3),

(1) Vienna Institute of Demography, Austrian Academy of Sciences (VID), Austria (2) Wittgenstein Centre of Demography and Global Human Capital (WIC), Austria (3) Institute of Statistics and Mathematical Methods in Economics, Research Unit Economics, Vienna University of Technology (TU), Austria

This paper investigates the differential impact that alternative pension systems have on the optimal years of schooling and retirement age for two groups of individuals that differ by their levels of life expectancy. Our analysis is implemented using a partial equilibrium model populated by overlapping generations, in which both groups –individuals with low and high life expectancy– interact through the pension system. Individuals endogenously choose their optimal number of years of schooling and retirement age through a life cycle model of labor supply. We complement our analysis by calibrating the model to the US pension system.

To separate out the interaction effects between the life expectancy and the pension system on the labor supply, we first analyze how an increasing gap between the life expectancy of both population groups has on the optimal number of years of schooling and the retirement age, for a fix set of parametric components of the pension system. Second, we study the influence that changes in the parametric components of the pension system have on the optimal number of years of schooling and retirement age of each group, assuming that the gap between their levels of life expectancy remain constant.

24. Differential games in environmental economics — 10:00 – 11:15

***Chair*: G. Zaccour** — **Böcklsaal**

The tragedy of ecosystems in open-access

Luc Doyen (1), Abdoul Cissé (2), Nicolas Sanz (3), Fabian Blanchard (2), Jean-Christophe Pereau (1),

(1) GREThA UMR CNRS, University of Bordeaux, Pessac, France (2) IFREMER, Domaine de Suzini, Cayenne, French Guiana, France (3) University of French Guiana, Cayenne, France

This paper investigates the role played by cooperation for the sustainable harvesting of an ecosystem. To achieve this, a bio-economic model based on a multi-species Gompertz dynamics with interspecific relationships and multi-agent catches is considered. A comparison between the non cooperative and cooperative optimal strategies is carried out. Revisiting the tragedy of open-access and over exploitation issues, it is first proved how harvesting pressure is larger in the non cooperative case for every species. Then it is examined to what extent gains of cooperation can also be derived for the state of the ecosystem. It turns out that cooperation clearly promotes the biodiversity when the number of agents is high. By contrast, when the number of agents remains limited, results are more ambiguous especially if a species by species viewpoint is adopted. However, an indicator is proposed at the ecosystem scale to highlight the gain of cooperation in the general case. Numerical examples illustrate the analytical findings.

Non-cooperative resource exploitation by patient players

Tapan Mitra (1), Gerhard Sorger (2),

(1) Department of Economics, Cornell University, Ithaca, NY, USA (2) Department of Economics, University of Vienna, Vienna, Austria

We consider a discrete-time dynamic game in which a finite number of players extract a non-renewable resource and derive consumption solely from the extracted amount (cake-eating game). Markov-perfect Nash equilibria can be constructed in this game not only if the players have time-preference factors that are smaller than 1, but also if these factors are equal to or even larger than 1. We demonstrate this result both for the case of identical players and for the case of heterogeneous players. In addition we study the influence of the model parameters on the equilibrium.

Pollution control in a multiregional setting: a differential game with spatially distributed controls

Javier de Frutos (1), Guiomar Martín-Herrán (1),

(1) IMUVA, Universidad de Valladolid, Spain

Most of the differential games modelling transboundary pollution problems neglect spatial or geographical relationships among agents (regions, countries, etc.). The main objective of this research is to analyze differential game models where pollution control is spatially distributed among a number, possibly large, of agents with predetermined spatial relationships. The analysis emphasizes the effects that could be expected as a consequence of the different geographical relationships among decision makers.

The differential game has one state variable (pollution stock) distributed among one large region divided in subregions which behave as decision agents controlling their own emissions of pollutants. The emissions are also represented as a distributed variable. We use the expression distributed variable to describe a function defined over the region or subregions under consideration. The dynamics of the state variable is defined by a convection-reaction-diffusion partial differential equation. As far as we know this type of problem has not received enough attention in the literature of differential games.

A first step in the analysis is to use aggregate variables (for example, the average in each subregion of each variable under consideration, pollution stock, emissions, etc.) in order to obtain models where the dynamics is represented by a system of ordinary differential equations with a large number of state variables and decision agents. In this type of models the geographical relationship can be represented by a graph that also determines the structure of the system of ordinary differential equations.

25. Set-valued and variational analysis 3 — 11:30 – 12:45

***Chair*: R. Cibulka** — **PrechtIsaal I**

Convergence theorems for iterative schemes based on metric regularity

Radek Cibulka (1), Asen L. Dontchev (2), Michel H. Geoffroy (3),

(1) NTIS - New Technologies for the Information Society and Department of Mathematics, Faculty of Applied Sciences, University of West Bohemia, Univerzitní 22, 306 14 Pilsen, Czech Republic (2) Mathematical Reviews, 416 Fourth Street, Ann Arbor, MI 48107-8604, USA (3) Laboratoire

de Mathématiques Informatique et Application (LAMIA), Département de Mathématiques, Université des Antilles et de la Guyane, F-97110 Pointe-à-Pitre, Guadeloupe (France)

Given Banach spaces X and Y, a single-valued *non-smooth* mapping $f : X \to Y$ and a multivalued mapping $F : X \rightrightarrows Y$, we study local convergence properties of the (in)exact Newton-type iterative schemes for solving the so-called generalized equation:

$$\text{Find} \quad x \in X \quad \text{such that} \quad 0 \in f(x) + F(x). \tag{1}$$

This model covers various problems such as equations, inequalities, variational inequalities, and, in particular, optimality conditions. The mapping f is approximated by a "generalized set-valued derivative" which in finite dimensions may be represented by Clarke's generalized Jacobian while in Banach spaces it may be identified with Ioffe's strict prederivative.

Based on various kinds of metric regularity, we intend to present Newton, Kantorovich, and Dennis–Moré theorems within the framework (1). As corollaries, we obtain results on convergence of inexact quasi-Newton type methods for semismooth equations. The presentation is based on the forthcoming papers [1,2,3].

[1] S. Adly, R. Cibulka, H. Van Ngai. Newton's method for solving inclusions using set-valued approximations. *SIAM Journal on Optimization*, to appear.

[2] R. Cibulka, A.L. Dontchev, M.H. Geoffroy. Inexact Newton methods and Dennis–Moré theorems for nonsmooth generalized equations. *SIAM Journal on Control and Optimization*, to appear.

[3] R. Cibulka, A.L. Dontchev, T. Roubal. Kantorovich-type theorem for non-smooth generalized equations. In preparation.

The Goh condition for closed control and endpoint constraints

Daniela Tonon (1), Hélène Frankowska (2),

(1) CEREMADE, Université Paris Dauphine, France (2) Institut de Mathématiques de Jussieu, UPMC Paris 6, France

We consider the Mayer optimal control problem in the presence of a closed control constraint and endpoint constraints. The Goh condition is known as a pointwise second order necessary optimality condition which turns out to be useful when the optimal control for the Mayer problem is singular. Indeed, in this case the classical Legendre-Clebsch condition is no more of use. We generalize the Goh condition to the case of a closed control constraint with very general properties (applicable not only to convex polytopes or constraints with smooth boundary), moreover we consider the case of endpoint constraints. New techniques are necessary in these cases.

On the projection processes with definable right-hand side

Nadezhda K. Ribarska (1, 2), Mira I. Bivas (1),

(1) Faculty of Mathematics and Informatics, University of Sofia, James Bourchier Boul. 5, 1164 Sofia, Bulgaria (2) Institute of Mathematics and Informatics, Bulgarian Academy of Sciences, Acad. G. Bonchev str., block 8, 1113 Sofia, Bulgaria

We study the relation between sweeping processes with the cone of limiting normals and projection processes. We prove existence of solution of a perturbed sweeping process with the cone of limiting normals and of nonstationary projection process, provided the sets involved are definable (and are moving in a definable way) in some o-minimal structure. An application to a crowd motion model is presented.

26. Infinite horizon optimal control and mathematical economics 1 11:30 – 12:45

***Chair*: A. Zaslavski** **Prechtlsaal II**

Adjoint variables and intertemporal prices in optimal economic growth problems

Sergey M. Aseev (1, 2),

(1) Steklov Mathematical Institute, Moscow, Russia (2) International Institute for Applied Systems Analysis, Laxenburg, Austria

The talk is devoted to properties of the adjoint variable in the relations of the Pontryagin maximum principle for a class of infinite-horizon optimal control problems arising in economic growth theory. Under appropriate growth or monotonicity assumptions (see [2] and [3] for details) the adjoint variable can be specified explicitly by a formula which is similar to the Cauchy formula for solutions of linear differential systems. We present an economic interpretation of the adjoint variable based on this formula and consider the formula's relationships to different transversality conditions at infinity. The discussed results are presented partly in [1].

[1] S.M. Aseev. On some properties of the adjoint variable in the relations of the Pontryagin maximum principle for optimal economic growth problems, *Proc. Steklov Inst. Math.*, 287(suppl. 1): 11–21, 2014.

[2] S.M. Aseev and A.V. Kryazhimskii. The Pontryagin maximum principle and optimal economic growth problems, *Proc. Steklov Inst. Math.*, 257(1): 1–255, 2007.

[3] S.M. Aseev and V.M. Veliov. Maximum principle for infinite-horizon optimal control problems under weak regularity assumptions. *Trudy Inst. Mat. i Mekh. UrO RAN*, 20(3): 41–57, 2014.

New results on infinite-dimensional discrete-times optimal control problems

Mohammed Bachir (1), Joël Blot (1),

(1) Laboratoire SAMM 4543, Universite Paris 1 Pantheon-Sorbonne, Centre P.M.F., 90 rue Tolbiac 75634, Paris, France

We present new results on necessary conditions of optimality for infinite horizon discrete-times optimal control problems when the state variables and control variables are infinite-dimensional. We us tools of linear and non linear analysis in Banach-spaces.

Special bounded solutions of infinite-horizon discrete-time optimal control problems

Joël Blot (1), Nhan Ngo Thoi (1),

(1) Laboratoire SAMM EA 4543, Université Paris 1 Panthéon-Sorbonne, Paris, France

We present new results on the behaviour of bounded solutions of infinite-horizon discrete-time optimal control problems around a turnpike process.

27. Health care 2 — 11:30 – 12:45

***Chair*: M. Kuhn, S. Wrzaczek** — **Festsaal**

An endogenous growth model with a health sector

Jesus Fernandez-Villaverde (2), Dirk Krueger (2), Alexander Ludwig (1), Matthias Schoen (3),

(1) SAFE, Goethe University Frankfurt; MEA; Netspar (2) University of Pennsylvania; CEPR; NBER (3) CMR, University of Cologne

In this paper we develop an Overlapping Generations model with endogenous growth and a health sector, in order to explain three secular facts characterizing the U.S. economy: a substantial increase in life expectancy, a rise in the share of GDP devoted to health-related expenditures as well as an increase in the relative price of medical goods. We show how to interpret these observations as the equilibrium outcome of a model in which technological progress is endogenously directed to the sector producing medical goods.

Growth and welfare under endogenous lifetime

Maik T. Schneider (1), Ralph Winkler (2),

(1) Department of Economics, University of Bath, UK (2) Department of Economics, University of Bern, Switzerland

In this paper, we study the relationship between endogenous investments in longevity, economic growth and welfare. For this purpose, we develop an endogenous growth model in which longevity is endogenously determined by the households' demand for healthcare services, and analyze how economic growth and welfare reacts to endogenous changes in life expectancy induced by improvements in the healthcare technology. We employ the simplest conceivable model bringing together all essential ingredients by combining the household side of perpetual youth models in the tradition of Blanchard (1985) with the production side of an endogenous

growth model in the style of Romer (1986) amended by a healthcare sector. To capture the welfare effects of increased longevity, we consider two different types of healthcare improvements. The first type decreases the baseline mortality, while the second type increases the marginal productivity of healthcare expenditures.

In our benchmark model specification, we find that improvements in the healthcare technology always lead to higher steady state growth rates. Intuitively, longevity influences the age structure of the economy. Higher longevity increases the relative share of old ("rich") to young ("poor") households and leads ceteris paribus to higher capital per capita accumulation. Our numerical calculations suggest that the effect on the growth rate is rather small. This lends support to the empirical literature on the welfare effects of longevity which abstracts from growth effects of increased longevity. However, for the welfare gains of increased longevity the type of healthcare improvements matters. For a given increase in longevity, welfare improvements are substantially higher if increased longevity is induced by improvements of the first type (baseline mortality decreases) compared to improvements of the second type (higher efficiency of healthcare treatment).

Our results for the growth effects of healthcare investments are very sensitive to the spillover specification in the production sector, which drives endogenous growth in our model. Generalizing the specification of the spillover effect, we show that longevity increases may lead to negative growth effects when technological improvements in the health sector trigger higher healthcare expenditures. Even small changes of the spillover specification yield negative growth effects which are consistent with real world data but involve welfare losses outweighing the welfare increase induced by higher longevity, and thus entail a negative overall welfare effect. By contrast, increases in expected lifetime enjoyed without healthcare expenditures always induce positive growth and welfare effects independent of the spillover specification.

It is obvious that equilibrium health expenditures cannot be efficient when endogenously determined increases in expected lifetimes result in long-run welfare losses. In fact, we identify two externalities associated with healthcare investments in our model. The first, which is well known, stems from the price taking behaviour of households with respect to annuities and leads to over-investment in healthcare. The second externality is a consequence of our growth model. In our benchmark model specification, households under-invest in healthcare, as they do not take into account that increased longevity on the aggregate level induces a positive effect on the economy's growth rate. We show that healthcare investments in the market equilibrium cannot be inefficiently high and give a condition under which they are inefficiently low. However, when the growth effect of increased healthcare investments turns negative in the generalized spillover specification, households over-invest rather than under-invest in healthcare services. Consequently, we derive conditions for which healthcare investments in the market equilibrium are inefficiently high. Thus, the nature of the growth effect of longevity also has direct implications for healthcare policy.

[1] P. Romer. Increasing returns and long-run growth. *Journal of Political Economy*, 94(5):1002–1037, 1986.

[2] O. Blanchard. Debt, deficits and finite horizons. *Journal of Political Economy*, 93(2):223–247, 1985.

A dynamic model of endogenous mortality, health investments and self-protection

Davide Dragone (1),

(1) Department of Economics, University of Bologna, Italy

In this paper I study a theoretical model where mortality is stochastic and endogenously affected by the health condition of the agent and by self-protection activities which do not directly affect health. I study the optimal trade-off between self-protection activities and health investments and I determine the impact of policies incentivizing either investments in health or self-protection choices.

The critical issue is predicting the long-run effects of these policy measures on individual behavior and mortality. Although the results depend on unobservables such as individual preferences and the shadow value of health and wealth, I show that it is possible to predict whether self-protection and investments in health will be long-run complements or substitutes, based on a short-run behavior that can be empirically observed, namely the individual change in behavior after a health shock.

If an individual increases her investment in self-protection activities right after receiving a negative health shock, then self-protection and health investments are predicted to be substitutes in the long-run. In such a case, policies aimed at promoting, say, self-prevention, lead to a compensatory decrease in health-investments and in health (and an ambiguous effect on mortality). If instead an individual reacts to a negative health shock by reducing investment in self-protection, in the long-run self-protection and health investments will be complements. Hence policies aimed at incentivizing self-prevention or health investments will unambiguously lead life-expectancy to increase.

28. Cooperation in differential games — 11:30 – 12:45

***Chair*: G. Zaccour** — **Böcklsaal**

Strongly time-consistent core in two or three-person cooperative differential games

Leon Petrosyan (1),

(1) Saint Petersburg State University, Saint Petersburg, Russia

Let $\Gamma(x_0, T-t_0)$ be the cooperative two or three-person differential game from the initial position x_0 with prescribed duration $T-t_0$, $\bar{x}(t)$ the corresponding cooperative trajectory maximizing the sum of players' payoffs, and $\Gamma(\bar{x}(t), T-t)$, $t \in [t_0, T]$ the family of subgames along cooperative trajectory. Suppose that the core $C(\bar{x}(t), T-t) \neq \varnothing$ for all $t \in [t_0, T]$ in subgames $\Gamma(\bar{x}(t), T-t)$. Let $\alpha \in C(x_0, T-t_0)$, and $\beta(\tau)$ be the IDP (imputation distribution procedure) for α, i.e., $\int_{t_0}^{T} \beta(\tau)d\tau = \alpha$. The core $C(x_0, T-t_0)$ is called strongly time-consistent if there exists such $\bar{\alpha} \in C(x_0, T-t_0)$ with corresponding IDP $\bar{\beta}(\tau)$, $\tau \in [t_0, T]$ that $\int_{t_0}^{t} \bar{\beta}(\tau)d\tau \oplus C(\bar{x}(t), T-t) \subset C(x_0, T-t_0)$ for $t \in [t_0, T]$ (here $a \oplus B = \{a+b : b \in B\}$ and $\bar{\alpha} = \int_{t_0}^{T} \bar{\beta}(\tau)d\tau$). It is proved that the core in two-person cooperative differential games (which in this case coincides with the set of all imputations) is strongly time-consistent, and under some

weak conditions it is also proved that the core is strongly time-consistent in three-person cooperative differential games.

Time-consistent dynamic bargaining and cooperative differential games

Jesús Marín-Solano (1), Jorge Navas (1),

(1) Department of Mathematical Economics, Universitat de Barcelona, Barcelona, Spain

We study cooperative solutions in differential games with heterogeneous agents, with different instantaneous utilities and/or discount rates. If players have equal discount rates, due to the lack of individual rationality (in general) along the whole time horizon of the Pareto efficient solutions, unless side payments among players are permitted, the sustainability of cooperation is not guaranteed. If players have different discount rates, the optimal solution maximizing the sum of intertemporal utilities becomes time-inconsistent. Time-consistent solutions can be found by looking for constrained Pareto efficient solutions at each instant of time by imposing as a constraint the future (time-consistent) behavior of the whole coalition. However, this solution can have important drawbacks, being the most important one that it can be strongly inefficient. In this work we abandon the Pareto efficiency condition by introducing nonconstant weights and search for time-consistent individually rational and group efficient cooperative solutions. These nonconstant weights can be obtained as the maximizers of a Nash welfare function. For different threat points, different time-consistent bargaining solutions can be derived. We apply our results to differential game models coming from the literature of natural resource economics.

A novel class of characteristic functions

Ekaterina V. Gromova (1), Leon Petrosyan (1),

(1) Faculty of Applied Mathematics and Control Processes, St.-Petersburg State University, St.-Petersburg, Russia

In game theory, a classical way to determine the characteristic function is to use *maxmin* formulation. firstly introduced in [1]. For differential games, it was proved [2] that this formulation yields a superadditive characteristic function. However, in the framework of differential games a maxmin solution is often difficult to find. Therefore, a number of alternative approaches has been presented, e.g., the Nash-based approach proposed in [3]. This approach has a drawback that the resulting characteristic function is not superadditive in general. In this contribution, we propose a novel way to find a superadditive characteristic function [4], based on the Pareto-optimal solution.

[1] Von Neumann, J., Morgenstern O., *Theory of Games and Economic Behavior*. Princeton University Press, 2007.

[2] Petrosyan, L. A., Danilov N. N., *Cooperative differential games and their applications*, Izd. Tomskogo University, Tomsk, 1982.

[3] Petrosjan L., Zaccour G. Time-consistent Shapley value allocation of pollution cost reduction *Journal of economic dynamics and control*, V.27(3): 381–398, 2003.

[4] Petrosyan L.A., Gromova E.V. Two-level cooperation in coalitional differential games, *Trudy Inst. Mat. i Mekh. UrO RAN*, 20(3):193–203, 2014.

29. Optimal control in finite and infinite dimension and application to economics 1

14:00 – 15:40

***Chair*: S. Faggian, F. Gozzi** **Prechtlsaal I**

Explicit investment rules with time-to-build and uncertainty

René Aïd (1), Salvatore Federico (2), Huyên Pham (3), Bertrand Villeneuve (4),

(1) EDF, Paris, France (2) DEMM, Università di Milano, Milano, Italy (3) LPMA, Université Paris 7, Paris, France (4) Université Paris Dauphine, Paris, France

We establish explicit socially optimal rules for an irreversible investment decision with time-to-build and uncertainty. Assuming a price sensitive demand function with a random intercept, we provide comparative statics and economic interpretations for three models of demand (arithmetic Brownian, geometric Brownian, and the Cox-Ingersoll-Ross). Committed capacity, that is, the installed capacity plus the investment in the pipeline, must never drop below the best predictor of future demand, minus two biases. The discounting bias takes into account the fact that investment is paid upfront for future use; the precautionary bias multiplies a type of risk aversion index by the local volatility. Relying on the analytical forms, we discuss in detail the economic effects. For example, the impact of volatility on the optimal investment is negligible in some cases. It vanishes in the CIR model for long delays, and in the GBM model for high discount rates.

Optimal investment and consumption in a delayed factor model

Cecilia Prosdocimi (1), Fausto Gozzi (1), Jun Sekine (2),

(1) Libera Università Internazionale degli Studi Sociali ”Guido Carli”, Rome, Italy (2) Osaka University, Japan

We study the optimal investment and consumption problem in a market governed by a factor, which lives in a Hilbert space and obeys to a stochastic functional differential equation. We are lead to solve a stochastic optimal control problem in infinite dimension. We characterize the value function and the optimal feedback strategies via duality methods. For CRRA utilities, the value function and the optimal strategies can be derived explicitly.

On the optimal exercise boundaries of swing put options

Tiziano De Angelis (1), Yerkin Kitapbayev (1),

(1) School of Mathematics, University of Manchester, UK

Swing options are financial instruments designed primarily to allow for flexibility on purchase, sale and delivery of commodities in energy markets. They have features of American-type options with multiple early exercise rights and may be mathematically described in terms of multiple optimal stopping problems.

In particular we consider a swing option with put payoff, N exercise rights and finite maturity, when the underlying asset's dynamics is described by a geometric Brownian motion according to the Black & Scholes model. We prove that the optimal exercise of each right (except the last) is characterised in terms of two free-boundaries which are continuous functions of time and uniquely solve a system of coupled integral equations of Volterra-type. Finally the option's price formula is provided as the sum of a European part and an early exercise premium depending on the optimal exercise boundaries.

Feynman-Kac representation of the value function for optimal control of pure jump Markov processes

Elena Bandini (1), Marco Fuhrman (1),

(1) Dipartimento di Matematica, Politecnico di Milano, Milano, Italy

We consider a classical finite horizon optimal control problem for continuous-time pure jump Markov processes described by means of a rate transition measure depending on a control parameter and controlled by a feedback law. For this class of problems the value function can often be described as the unique solution to the corresponding Hamilton-Jacobi-Bellman equation. We prove a probabilistic representation for the value function, known as nonlinear Feynman-Kac formula. It relates the value function with a backward stochastic differential equation (BSDE) driven by a random measure and with a sign constraint on its martingale part. We also prove existence and uniqueness results for this class of constrained BSDEs. The connection of the control problem with the constrained BSDE uses a control randomization method recently developed in the works of I. Kharroubi and H. Pham and their co-authors. This approach also allows to prove that the value function of the original non-dominated control problem coincides with the value function of an auxiliary dominated control problem, expressed in terms of equivalent changes of probability measures.

30. Infinite horizon optimal control and mathematical economics 2 14:00 – 15:40

***Chair*: A. Zaslavski** **PrechtlsaaI II**

Viability problems in economic theory

Toru Maruyama (1),

(1) Keio University, Tokyo, Japan

Let Ω be a subset of a Banach space $\mathfrak{X}$, and $A:\ \Omega \to \mathfrak{X}$ a continuous function. Consider an ordinary differential equation

$$u' = Au, \quad u(t_0) = x_0\ (\in \Omega). \tag{1}$$

If there exists some solution $u(t)$ of (1) which remains in Ω on some time-interval $[0,T]$, i.e. $u(t) \in \Omega$ for $t \in [0,T]$, $u(t)$ is called a **viable solution** of (1) on $[0,T]$.

When we study various dynamic problems in economic theory (say, price adjustments, trade cycles, and so on), we have to start with the existence of a viable solution of (*) in the positive cone in $\mathbb{R}^l$ on $[0,\infty)$. We can proceed to further examinations of the dynamic properties of the processes only after this viability problem is cleared.

The pupose of my talk is to give coherent foundations of this problem and to show how to apply them to concrete economic theories.

Infinite-horizon multiobjective optimal control problems under constraints

Naila Hayek (1),

(1) CRED EA 7321, Université Panthéon-Assas Paris 2, Paris, France

Multiobjective optimal control problems with constraints are studied for systems governed by difference equations in the discrete-time case and infinite-horizon framework. A Pontryagin Maximum Principle in the strong form is established by using Alternative theorems and a generalization of results due to Philippe Michel notably a generalization of the notion of mixed problems to multiobjective optimization.

[1] N. Hayek. A generalization of mixed problems with an application to multiobjective optimal control. *Journal of Optimization Theory and Applications*, 150(3), 498-515, 2011.

[2] J. Blot, N. Hayek *Infinite-horizon optimal control in the discrete-time framework*. SpringerBriefs in Optimization, Springer, New York, 2014.

A reverse-calculation method to utility from the policy function for optimal growth models

Yuhki Hosoya (1),

(1) Kanto-Gakuin University, Japan

This study constructs a method for calculating the utility function and discount rate of the simplest discrete-time Ramsey-Cass-Koopmans model from the policy and production functions, and provide a necessary and sufficient conditions for the policy function which can be interpreted by such a model. Further, this study shows the similar result in the Ramsey-Cass-Koopmans model with stochastic shock or endogenous labor supply.

Adaptation, mitigation and risk: an analytic approach.

Amos Zemel (1),

(1) The Blaustein Institutes for Desert Research, Ben Gurion University of the Negev, Sede Boker Campus, Israel

The adaptation vs. mitigation dilemma is considered by analyzing a simple dynamic model of managing a polluting process subject to the risk of abrupt occurrences of harmful events. The occurrence hazard can be mitigated by reducing the polluting emissions, while the occurrence damage can be controlled via investments in adaptation activities. The problem involves a multi-state, multi-control formulation. A full dynamic characterization of the optimal mitigation/adaptation processes is presented. The adaptation and mitigation policies interact strongly, and the evolution in time of each is affected by the option to implement the other. The conditions under which adaptation investments should begin promptly, take place after some delay or be avoided altogether are derived in terms of some key model parameters.

31. Dynamic games models of institutional change — 14:00 – 15:40

***Chair*: R. Boucekkine** — **Festsaal**

The warrior's choice: plunder, tribute, peace or conquest

Kenneth S. Chan (1), Jean-Pierre Laffargue (2),

(1) Department of Economics, McMaster University, Canada (2) Department of Economics, University of Paris 1, France

We develop a discrete-time dynamic model of a producer and a warrior country. The part of the output of the first country, which is not seized by the second one, is allocated between consumption and military defense. The dynamics of the population of the first country (the state variable in the dynamic game) depends on its consumption, according to a Malthusian framework. The warrior country allocates its output between consumption and military attack power. In each period, the equilibrium of the model is of one of the four following types:

- Either the warrior country attacks and plunders the producer country. It also sets its military expenditure and the other country reacts by deciding how much resource it devotes to defense.
- Or the warrior country negotiates the payment of a tribute by the producer country. Then, the military expenditures of both countries are zero.
- Or the cost of aggression is higher than its benefits and the warrior country opts for peace. The military expenditures of both countries are zero.
- Or these three kinds of anarchy equilibrium break down and the warrior country conquers the producer country. Then, it will have to permanently spend on military but also can tax the production of its conquest.

We compute the stagewise feedback Stackelberg equilibrium of the dynamic game and investigate how the structural features of the two countries determine which equilibrium prevails. The theoretical results are illustrated by historical examples from medieval Europe and Imperial China.

Shall fiscal policies be centralized in a monetary union? A dynamic game approach

Reinhard Neck (1), Dmitri Blueschke (1),

(1) Department of Economics, Alpen-Adria Universität Klagenfurt, Austria

In this paper we present an application of the dynamic tracking games framework to a monetary union. We use a small stylized nonlinear two-country macroeconomic model of a monetary union for analysing the interactions between fiscal (governments) and monetary (common central bank) policy makers, assuming different objective functions of these decision makers. Using the OPTGAME algorithm we calculate solutions for two game strategies: one cooperative (Pareto optimal) and one non-cooperative game type (the Nash game for the feedback information pattern). Applying the OPTGAME algorithm to the MUMOD1 model we show how the policy makers react upon demand shocks according to these solution concepts. To this end we introduce two sequences of different demand side shocks on the monetary union. The first sequence includes a negative asymmetric demand side shock aimed at describing the dynamics in a monetary union in a situation similar to the economic crisis (2007-2010) and the sovereign debt crisis (since 2010) in Europe. The second sequence of the shocks includes different kinds of demand side shocks and serves to discuss the best macroeconomic policy strategies for possible future scenarios. In particular, we investigate the welfare consequences of two scenarios: decentralized fiscal policies by independent governments (the present situation), both under a non-cooperative and a cooperative mood of play, and a centralized fiscal policy under different assumptions about the joint objective function corresponding to different weights for the two governments in the bargaining process assumed to precede the design of the common fiscal policy. We show the crucial importance of these weights (and hence of the regulations contained in the fiscal constitution of the union) for the results of the outcome in terms of sustainability of fiscal policies and main objective variables of the policy makers.

The political economy of labor market regulation with R&D

Tapio Palokangas (1,2,3),

(1) HECER, University of Helsinki, Helsinki, Finland (2) IIASA, Laxenburg, Austria (3) IZA, Bonn, Germany

This paper examines the political rationale for labor market regulation. In the high-tech sector, self-interested policy makers regulate local labor markets, R&D firms employ high-tech labor, oligopolists employ high-tech and ordinary labor and bargain over wages with the latter, and employer and labor lobbies influence the policy makers by their political contributions. The competitive sector employs only raw labor. It is shown that the empirically observed tendency to labor market deregulation results from decentralized labor market policy. With R&D, there is a welfare-maximizing number of jurisdictions with a different policy maker.

[1] T. Palokangas. The Political Economy of Labor Market Regulation with R&D. *IZA Discussion Paper* No. 8147, 2014.

Differential games with (a)symmetric players and heterogeneous strategies

Benteng Zou (1),

(1) CREA, University of Luxemourg, Luxembourg

One family of heterogeneous strategies in differential games with (a)symmetric players is developed in which one player adopts an anticipating open-loop strategy and the other adopts a standard Markovian strategy. Via conjecturing principle, the anticipating open-loop strategic player plans his strategy based on the possible updating the rival player may take. These asymmetric strategies frame nondegenerate Markovian Nash Equilibrium, which can be subgame perfect. Except the stationary path, this kind of strategy makes the study of short-run trajectory possible, which usually are not subgame perfect. However, the short-run nonperfection provides very important policy suggestions.

32. Dynamic games in economics and business 2 — 14:00 – 15:40

***Chair*: S. Chair** — **Böcklsaal**

Multi-agent experimentation on a common threshold

Yi Chen (1),

(1) Economics Department, Yale University, United States

A multi-agent dynamic game of experimentation is examined where players noncooperatively search for a common unknown threshold. Time is discrete and players take turns in adjusting their individual level of performance. There is a common threshold of performance below which a player suffers a (lump sum) punishment. Information is shared by all, and players start with a common prior with regard to the distribution of the threshold.

For time intervals that are sufficiently short, there always exists a pure-strategy MPE. Closed-form descriptions are obtained for equilibrium strategy, value, and time path of performance level in the limit as the interval length tends to zero. In equilibrium, learning is gradual and eventual learning is not guaranteed. There is an asymptotic level of performance where learning stops in the long run. The dynamics of the multi-agent game pose a sharp contrast to that of a single-agent decision problem, because in the latter, the level of performance declines to the asymptotic level almost instantly when the time interval is short. The decline in performance is slower with more players.

Bounded rationality, learning-by-doing and Cournot competition in a stochastic environment

Konstantin Kogan (1), Fouad El Ouardighi (2),

(1) Management Department, Bar-Ilan University, Faculty of Social Sciences, Ramat-Gan, Israel
(2) Operations Management Department, ESSEC Business School, Cergy Pontoise, France

A decrease in unit production cost due to cumulative production experience - “learning by doing” is extensively observed in practice. It has been shown that under mature technologies learning curves exhibit a linear behavior in cumulative production. In this paper we consider two competing firms characterized by linear dynamics of learning by doing with respect to their unit production cost. We assume production experience is affected by random causes independent of the number of products produced and the process may involve costless spillovers of learning by the competing firm. Similar to the Cournot competition, in this differential game the firms compete by choosing their outputs in terms of fully substitutable products. Their choice defines the inverse demand, i.e., the market price of the products and thereby their profits. Unlike the Cournot assumption that firms maximize their profits taking as given the quantity of the rival firms, we assume that the firms are not necessarily fully rational. Furthermore, they may not be able to determine their competitor’s reaction to change in their output. As a result, they conjecture such a response. Comparing open- as well as closed-loop equilibrium behavior of the fully rational competing firms with that based on consistent conjectural variations we provide an insight on how production rate and accumulated experience are affected depending on the level of learning spillovers, product life time and customer price sensitivity.

On the long-run survival of socially concerned firms under evolutionary pressure

Michael Kopel (1), Fabio Lamantia (2),

(1) Department of Organization and Economics of Institutions, University of Graz, Austria
(2) Department of Economics, Statistics and Finance, University of Calabria, Italy

We study an oligopoly market where profit-maximizing firms and socially concerned firms compete in quantities. First, we characterize mixed industry equilibria in terms of unit cost differentials. Second, we give conditions based on cartel stability for such mixed equilibria to occur. We then introduce an evolutionary setting to investigate the endogenous choice of the proper objective of business firms and the influence of unit cost differences on the long run survival of firms which pursue non-profit motives. Based on a strategy’s profitability, we consider evolutionary updating with firms selecting Nash quantities or choosing best replies to the expected market quantity (see Kopel, Lamantia, and Szidarovszky 2014). The novelty of the present paper is that in every period N firms (instead of just 2) are matched to play the oligopoly game. We describe the long-run outcomes of the evolutionary setting and compare the stability conditions with the conditions for cartel stability.

[1] Kopel, M. and F. Lamantia, F. Szidarovszky (2014), Evolutionary competition in a mixed market with socially concerned firms, *Journal of Economic Dynamics & Control*, 48, 394–409.

On determining Nash equilibria for stochastic positional games

Dmitrii Lozovanu (1), Stefan Pickl (2),

(1) Institute of Mathematics and Informatics of Moldova Academy of Sciences, Moldova

(2) Institute for Theoretical Computer Science, Mathematics and Operations Research, Universität der Bundeswehr München, Germany

We consider a class of stochastic positional games that extends deterministic positional games and Markov decision processes with average and expected total discounted costs optimization criteria (see [1,2]). Necessary and sufficient conditions for the existence of Nash equilibria in the considered class of games are proven. Based on a constructive proof of these results and the results from [1,2] we propose new approaches for determining the optimal strategies of the players. The computational complexity of the problem of determining Nash equilibria in stochastic positional games with average and discounted payoffs will be analysed. Furthermore we show that the obtained results can be used for studying the problem of existence of Nash equilibrium in Shapley stochastic games with average and discounted payoffs.

[1] D. Lozovanu, S. Pickl, E. Kropat. Markov decision processes and determining Nash equilibria for Stochastic positional games. *In: Proceedings of the 18th World Congress IFAC-2011*, 13398-13493, 2011.

[2] D. Lozovanu, S.Pickl,. *Optimization of Stochastic Discrete Systems and Control on Complex Networks*. Springer, 2015.

33. Optimal control in finite and infinite dimension and application to economics 2

16:00 – 17:40

***Chair*: S. Faggian, F. Gozzi** **Prechtlsaal I**

Short-run pain, long-run gain: the conditional welfare gains from international financial integration

Raouf Boucekkine (1), Giorgio Fabbri (2), Patrick A. Pintus (2),

(1) Aix-Marseille School of Economics, Marseille, France (2) EPEE, Université d'Evry-Val-d'Essonne, Département d'Economie,Evry, France (3) Aix-Marseille School of Economics, Marseille, France

This paper aims at clarifying the conditions under which financial globalization originates welfare gains in a simple endogenous growth setting. We focus on the capital-deepening effect of financial globalization in an open-economy AK model and we show that constrained borrowing triggers substantial welfare gains, even at small levels of international financial integration, provided that the autarkic growth rate is larger than the world interest rate. Such conditional welfare benefits boosted by stronger growth - long-run gain - arise in our preferred model without investment commitment and they range, relative to autarky, from about 2% in middle-income countries to about 13% in OECD-type countries under international financial integration. Sizeable benefits emerge despite the fact that consumption initially falls - short-run pain - and that welfare-reducing growth breaks materialize when the economy switches from autarky to financial integration, which is however shown not to dwarf positive welfare changes.

Optimal control of Varicella and Herpes Zoster by Varicella vaccination only: the effects of immunity boosting

Monica Betta (1), Marco Laurino (2), Andrea Pugliese (3), Giorgio Guzzetta (4), Alberto Landi (1), Piero Manfredi (5),

(1) Department of Information Engineering (DIE), University of Pisa, Italy (2) Institute of Life Science, Scuola Superiore Sant'Anna, Pisa, Italy (3) Department of Mathematics, University of Trento, Italy (4) Bruno Kessler Foundation, Trento, Italy (5) Department of Statistics and Mathematics Applied to Economics, University of Pisa, Italy

A main obstacle to the widespread adoption of varicella immunization in Europe has been the fear of a subsequent boom in natural herpes zoster caused by the decline in the protective effect of natural immunity boosting due to reduced virus circulation. We apply optimal control to simple models for VZV transmission and reactivation to investigate existence and feasibility of temporal paths of varicella childhood immunization that are optimal in controlling both varicella and zoster. We analyze the optimality system numerically focusing on the role played by the structure of the cost functional, the relative cost zoster-varicella, and the length of the planning horizon. We show that optimal programs exist but will mostly be unfeasible in real public health contexts due to their complex temporal profiles. This complexity is the consequence of the intrinsically antagonistic nature of varicella immunization programs when aimed to control both varicella and herpes zoster. However we could show that gradually increasing, smooth thereby feasible - vaccination schedules, can perform largely better than routine programs with constant vaccine uptake. Moreover we show the optimal temporal profiles of feasible immunization programs targeting with priority the mitigation of the post-immunization natural zoster boom.

On the dynamic programming approach to optimal control of delay equations with delay in the control, a deterministic case

Mauro Bambi (1), Cristina Di Girolami (2), Salvatore Federico (3), Fausto Gozzi (4),

(1) University of York, Department of Economics and Related Studies, York, UK (2) Università di Chieti Pescara, Dipartimento di Economia Aziendale, Pescara, Italy (3) Università degli Studi di Milano, Dipartimento di Scienze Economiche Aziendali e Statistiche, Milan, Italy (4) Università LUISS - Guido Carli, Dipartimento di Scienze Economiche ed Aziendali, Rome, Italy

We consider an optimal control problem where the state equation is a delay equation with delay in the control. The problem is restated as an optimal control problem in infinite dimension. We treat it by the dynamic programming approach studying the existence, uniqueness and regularity of solutions of the associated HJB equation. Then a verification theorem is given. This class of problems arises in some economic applications, in particular in problems of optimal growth with time-to-build. A deterministic case will be studied in details In this paper we suppose flexible investment projects. The concept of investment project was introduced for the first time in a partial equilibrium framework by Lucas and later in a general equilibrium model by Kydland and Prescott to allow for gestation lags in the production of capital goods. A project, as defined by these authors, requires several stages before its completion and then new productive capital needs several periods to be built and used. Moreover the resources allocated to a project as well

as its objective have to be decided at the beginning of the project itself and cannot be adjusted afterward. This feature seems quite restrictive once compared with the empirical evidences. The main part of this paper presents an endogenous growth model with flexible multi-period investment projects. The engine of growth in our economy is the presence of constant returns to scale in the capital stock which is the only accumulating factor of production. We write the optimal control problem in a suitable Hilbert space. The associated Hamilton- Jacobi-Bellman (HJB) equation is considered in this space. We explicitly find the value function, the closed loop formula that relates capital and investment, the optimal consumption path and the long run equilibrium.

Optimal control of an SIR epidemic model with changing behavior through education campaign

Sanjukta Hota (1), Hem Raj Joshi (2), Suzanne Lenhart (3), Folashade Agusto (4),

(1) Dept. of Mathematics, Fisk University, Nashville, USA (2) Dept. of Mathematics, Xavier University, Cincinnati, USA (3) Dept. of Mathematics, University of Tennessee, Knoxville, USA (4) Dept. of Mathematics, Austin Peay State University, Clarksville, USA

An SIR type epidemic model is expanded to include the use of education or information given to the public as a control to manage a disease outbreak when effective treatments or vaccines are not readily available or too costly to be widely used. We study stability analysis and use optimal control theory on the system of differential equations to achieve the goal of minimizing the infected population. We illustrate our results with some numerical simulations.

34. PDE-constrained optimization 2 — 16:00 – 17:40

***Chair*: I. Neitzel, A. Rösch** — **PrechtlsaaI II**

Error estimates for optimal control problems related to the Navier-Stokes equations

Konstantinos Chrysafinos (1), Eduardo Casas (2),

(1) Department of Mathematics, National Technical University of Athens, Athens, Greece
(2) Departamento de Matemática Aplicada y Ciencias de la Computación, Universidad de Cantabria, Santander, Spain

We discuss a-priori error estimates for the velocity tracking of three-dimensional evolutionary Navier-Stokes flows. The controls are of distributed type, and subject to point-wise control constraints. The discretization scheme of the state and adjoint equations is based on a discontinuous time-stepping scheme (in time) combined with conforming finite elements (in space) for the velocity and pressure. Provided that the time and space discretization parameters, τ and h respectively, satisfy $\tau \leq Ch^2$, error estimates of order $\mathcal{O}(h)$ depending on the regularity of the target and the initial velocity, are proved for the difference between the locally optimal controls and their discrete approximations. Both results are based on construction of suitable space-time projections, for the evolutionary Navier-Stokes equations.

Second-order analysis for sparse control of FitzHugh-Nagumo equations and application

Eduardo Casas (1), Christopher Ryll (2), Fredi Tröltzsch (2),

(1) Department of Applied Mathematics and Computer Science, University of Cantabria, Santander, Spain (2) Institut für Mathematik, Technische Universität Berlin, Berlin, Germany

We investigate the problem of sparse optimal controls for the so-called FitzHugh-Nagumo system. The solution of this reaction-diffusion equation forms, for certain settings of parameters, patterns of spiral waves. Sparsity is the consequence of the presence of the L^1-norm of the control in the objective functional. We refer to [1], where the first order necessary optimality condition for the mentioned problem is derived. Moreover, some numerical examples are given. In [1], a Tykhonov regularization term is used. This term is usually weighted with a small constant $\kappa > 0$. We observed in our examples that even a very small $\kappa > 0$ does not influence the numerical stability. Therefore we investigate the convergence analysis for $\kappa \to 0$. To this aim, following [2], we derive a first order necessary optimality condition for $\kappa = 0$ and second order sufficient optimality conditions for the optimal control of the FitzHugh-Nagumo system not only for $\kappa > 0$, but also for $\kappa = 0$. Finally, also the convergences of the control and state variables are proved for passing to the limit $\kappa \to 0$.

[1] E. Casas, C. Ryll, F. Tröltzsch. Sparse optimal control of the Schlögl and FitzHugh-Nagumo systems *Computational Methods in Applied Mathematics*, 13(4), 415–442, 2013.

[2] E. Casas, F. Tröltzsch. Second-Order and Stability Analysis for State-Constrained Elliptic Optimal Control Problems with Sparse Controls *SIAM J. Control Optim.*, 52(2), 1010–1033, 2014.

Finite element analysis of Free Material Optimization problems

Michael Hinze (1), Tobias Jordan (1),

(1) Schwerpunkt Optimierung und Approximation, Universität Hamburg, Germany

In Free Material Optimization (cf., e.g., [2]) the design variable is the full material tensor of an elastic body. Written in matrix notation one obtains a control-in-the-coefficients problem for the material tensor.

In this talk we will present recent results in the finite element analysis in Free Material Optimization, inspired by techniques used in the work [1] to identify the diffusion matrix of an elliptic problem. We employ the variational discretization approach, where the material tensor, which acts as the control variable, is only implicitly discretized. Numerical examples supplement our analytical findings.

[1] K. Deckelnick and M. Hinze. Convergence and error analysis of a numerical method for the identification of matrix parameters in elliptic PDEs. *Inverse Problems*, 28(11):115015, 2012.

[2] G. Leugering, M. Stingl. PDE-constrained optimization for advanced materials. *GAMM-Mitteilungen*, 33(2):209–229, 2010.

Optimal control of a coupled ODE-PDE model of a truck with a fluid basin

Sven-Joachim Kimmerle (1), Matthias Gerdts (1),

(1) Departemnt of Mathematics and Computer Application, Universität der Bundeswehr München, Neubiberg/Munich, Germany

We consider an optimal control problem for a truck with a fluid basin as load, which leads to an optimal control problem with a coupled system of partial differential equations (PDEs) and ordinary differential equations (ODEs).
The motion of the fluid in the basin is modelled by the nonlinear hyperbolic Saint-Venant (shallow water) equations while the vehicle dynamics are described by Newton's equations of motion. The fluid basin is fixed to the truck by a spring-damper system. The PDEs and ODEs are fully coupled through boundary conditions and force terms. The total system may be controlled by the acceleration of the truck.
We follow a first-discretize-then-optimize approach using a Lax-Friedrich scheme and present numerical results for optimal braking maneuvers, the objective including a time-optimal and a tracking-type ("safety") term. Furthermore, we discuss necessary optimality conditions.

[1] M. Gerdts and S.-J. Kimmerle. Numerical optimal control of a coupled ODE-PDE model of a truck with a water basin. Submitted to *AIMS proceedings*, 2014.

35. Economics of climate change — 16:00 – 17:40

***Chair*: W. Semmler** — **Festsaal**

Environmental pollution in a growing economy with endogenous structural change

Anton A. Bondarev (1), Alfred Greiner (2),

(1) Department of Business Administration and Economics, University of Basel, Switzerland
(2) Department of Business Administration and Economics, University of Bielefeld, Germany

In this paper we study the impact of environmental pollution in an endogenous growth model with structural change. The paper allows for both horizontal and vertical innovations where newer, less polluting technologies gradually replace older ones. All new technologies are homogeneous in their investment characteristics but higher abundance of assets leads to faster development of newer technologies. Since the labour in limited, it is redistributed from older sectors to newer ones, causing structural change.

The analysis shows that the presence of environmental externalities stimulates structural change but reduces the growth rate of the economy. Further, comparing the models with and without structural change demonstrates that the latter implies stronger environmental damages and, consequently, a lower growth rate than the first one. Finally, levying a tax on the polluting output speeds up structural change, thus, reducing environmental pollution and spurring economic growth.

Terminal peak in exhaustible resource use

Hagen Schwerin (1),

(1) ETH Zurich, Switzerland

Energy derived from fossil resources coal, oil, and natural gas has persistently grown—on average by 3.37% annually since 1875 when it satisfied 1/6 of world energy demand until 2000 when it provided 4/5 of demand, and so did tons of coal, barrels of oil, and cubic feet of natural gas. This 40-fold increase compares to the meager doubling of the sum of renewable and nuclear energy over the same time span. In this paper, I construct a vintage capital model that yields a novel prediction about peaking exhaustible resource use. Fossil energy peaks at the latest date of its investment absent an energy technology revolution and climate policy, when at future dates renewable energy technology is sufficiently productive at large scale to sustain consumption growth. The model is estimated and solved to explain the development of fossil and renewable energy 1800-2000. Simulations of the model predict at what date in the future the use of exhaustible fossil energy will peak and when it will be abandoned both with and without allocative effects of carbon tax policy. The terminal peak of fossil energy has important policy implications. A technology revolution that advances renewable energy over fossil energy at all scales will put an immediate stop to fossil energy use. Therefore, in anticipation of such an event and while it has not occured yet, fossil energy use more rapidly increases when such an event is more likely to occur. The novelty of this “Green Paradox” result is that it is derived with endogenous technology. As a result, scientific research making the technology revolution more likely to occur is counterproductive for the aim of reducing carbon dioxide emissions in the short-term. Climate policy shifts the peak of fossil energy toward the present and extends the period of using fossil energy. This is in sharp contrast to the literature on fossil energy use predicting that it declines in equilibrium. I find that fossil fuel use is capped in the 2080’s to limit atmospheric carbon dioxide by 720 ppmv.

Dynamic game of international environmental cooperation

Marita Laukkanen (1), Yulia Pavlova (2),

(1) VATT, Governmental Center for Economic Research, Finland (2) Natural Resource Institute Finland (LUKE), Finland

This paper analyzes a dynamic model of international environmental cooperation, where the stock of pollution affects both signatory and non-signatory countries’ emissions, and the stability of the IEA. The closest to our paper is a paper by Rubio and Ulph [1], where membership of the IEA is allowed to vary depending on the evolution of the stock. [1] concludes that the prospects for cooperation are not very bright: the size of the stable coalition decreases as the emission stock approaches its steady state.

Even though Rubio and Ulph’s paper is an important step towards analyzing countries strategic incentives for participation in an environmental treaty, the model in [1] is of limited applicability in the majority of real international and regional settings. A number of strong assumptions are made in order to achieve analytical result. For instance, linear benefits from emissions are unlikely to find empirical support. Furthermore, the model can only be applied to cases where the

number of potential participants is large (at least 20-30 countries), which does not hold in the case of local pollutants and regional treaties. Finally, numerical illustrations in [1] concern the case where the current pollution stock is below the cooperative steady state, whereas in real world the opposite is more likely.

The present paper adopts the more realistic work-horse model of IEAs where countries' per period payoffs are quadratic in emissions (similar static version of the model can be found e.g. in [2], [3]). Optimal emissions and endogeneous IEA membership are thus modelled in a linear-quadratic framework. Stable coalition is determined through internal/external stability rule at every moment. We consider two different approaches to dynamics of IEA membership and apply the model to a numerical calibration reflecting the climate policy problem. We find that when the dynamic dimensions of international environmental cooperation are accounted for, the grand coalition can be supported as an outcome of the game for a range of parameter values.

[1] S. Rubio, and A. Ulph. An infinite-horizon model of dynamic membership of international environmental agreements. *Journal of environmental economics and management*, 54(3):296–310, 2007.

[2] S. Barrett. Self-enforcing international environmental agreements. *Oxford Economic Papers*, 46:878–894, 1994.

[3] Y. Pavlova, and A. J. de Zeeuw. Asymmetries in international environmental agreements. *Environmental and Development Economics*, 18(1):51–68, 2013.

An integrated assessment model with mitigation and adaptation as climate policies

Helmut Maurer (1), Willi Semmler (2), Stephan Klasen (3), Tony Bonen (2),

(1) University of Muenster, Münster, Germany (2) New School for Social Research, New York, USA (3) University of Goettingen, Göttingen, Germany

Public policy to ameliorate the negative effects of climate change requires investments in both the mitigation of future CO2 emissions and adaptation against climate change damages. This paper explores analytically the interplay of growth, mitigation, and adaptation through an integrated assessment model (IAM) in which CO2 emissions are driven by the use of a non-renewable resource such a fossil energy. Policies to mitigate climate change include efforts to reduce CO2 emission as well phasing in renewable energy. Policies to adapt to climate change includes the provision of infrastructure against climate risk. The optimal policies of resource extraction, mitigation, adaption and infrastructure investment as well as the decisions on tax revenues and consumption are studied. The high dimensional model in terms of state and decision variables are analytical as well as numerical explored. Numerically the model is approximated with finite decision horizon using nonlinear model predictive control (NMPC) and the AMPL algorithm.

36. Harvesting 1 — 16:00 – 17:40

***Chair*: S. Behringer, T. Upmann** — **Böcklsaal**

Is aquaculture really an option?

Esther Regnier (1), Katheline Schubert (1),

(1) Paris School of Economics, University Paris 1, France

This article analyzes the impact of the introduction of aquaculture on wild fish stocks and consumer utility, taking into account three key components: (1) the dependence of aquaculture on reduction fisheries for the feeding of the farmed species; (2) biological interactions between the wild edible species –the predator– and the wild feed species –the prey–; (3) consumer preferences on wild and farmed fish. The wild fisheries are in open access while the aquaculture sector is competitive. We show that when biological interactions are moderate, the introduction of aquaculture is beneficial in the long run: it improves consumer utility and alleviates the pressure on the edible fish stock. The results are deeply modified when biological interactions are strong: the stock of edible wild fish is reduced and the introduction of aquaculture may even be the cause of a decrease of consumer utility. Finally, we explore the consequences of an improvement of aquaculture efficiency and of a sensitivity of consumer preferences to the farmed fish type, in the case where biological interactions are absent.

Extraction of timber and biomass for energy in size distributed forests under climate change

Angels Xabadia (1), Joan Pujol (1), Renan U. Goetz (1),

(1) University of Girona, Girona, Spain

Moderation of climate change and the adaptation to its effects is one of the most important challenges that humanity faces in the twenty-first century. Climate change affects many natural resources, and forest ecosystems are not an exception. It is generally accepted that the increase in atmospheric CO_2 acts as a fertilizer and, within certain limits, promotes forest growth. However, the increase in temperature and changes in the overall precipitations as well as their seasonal distribution will likely lead to an increase in forest mortality in southern Europe. This is specially important in Spain, where in many regions the low profitability of forests has led to the discontinuation of forests activities.

One way to increase profitability of forests could be the extraction of forest biomass as an additional forest product to be used as an energy source. Biomass has the potential to become a major source of renewable energy, and it is considered that integrated bioenergy systems will be important to ensure the future supply of energy. Moreover, the exploitation of the forest biomass for energy purposes could replace part of the energy consumption from non-renewable sources, thus moderating the climate change. The objective of the paper is to determine the adaptation of the management regime in size-structured forests to take account of this possibility and to assess its future profitability.

Optimal control problems for some diffusive processes. Geometric properties of the support of the optimal control

Sebastian Aniţa (1), Ana-Maria Moşneagu (1),

(1) Faculty of Mathematics, Alexandru Ioan Cuza University of Iaşi, Bd. Carol I nr. 11, Iaşi 700506, Romania

We investigate some internal optimal control problems for diffusive processes. We derive necessary optimality conditions for the situation when the support of the control is fixed and use them to get information concerning the best position and shape for the support of the control. We focus on both aspects: the optimal magnitude of the control and the optimal position of the support of the control.

A special attention is given to the optimal harvesting problems. We derive an iterative algorithm which improves at each step the shape and position of the support of the harvesting effort.

Social status, envy in common pool renewable resources

Hassan Benchekroun (1), Ngo Van Long (1),

(1) McGill University, Montreal, Canada

We examine the impact of social status in a common pool renewable resource oligopoly. A small number of players share access to a common pool resource and sell their production in a common market where they are oligopolists.

We depart from the main literature on common pool resource oligopolies by considering that each player cares about her social status. We allow for two channels to impact a player's welfare: harvest and profits. Under the first channel, a player has a bump in her utility when her harvest is larger than the average harvest of the rest of the players. In this case we show that the presence of this channel exacerbates the tragedy of the common and in a symmetric game all agents are worse off than if they did not compare their harvest to the harvest of the other players. Under the second channel, a player enjoys a bump in her utility if she manages to earn more profits than the average profits of the other players. In this case we show that social status may alleviate the tragedy of the commons: result in a decrease of extraction and increase in welfare compared to the case where social status considerations are absent.

Existence of optimal stationary state of exploited population with asymmetric intraspecific competition

Alexey A. Davydov (1,2,3), Amer F. Nassar (1),

(1) Lomonosov Moscow State University, Moscow, Russia (2) Vladimir State University, Vladimir, Russia (3) International Institute for Applied Systems Analysis, Vienna, Austria

We consider dynamic of size structured population which is described by the equation

$$x_t(t,l)+[g(l,E(t,l))x(t,l)]_l = -[\mu(l,E(t,l))+u(l)]x(t,l),$$

where $x(t,l)$ is the density of individuals of size l at the moment t; g and μ are growth and death rates, respectively, and u stays for exploitation intensity. The competition level E is of the asymmetric form $E(t,l) = \int_l^L \chi(l)x(t,l)dl$ with some nonnegative integrable function χ on interval $[0,L], L > 0$, where we manage and exploit the population. The inflow of new individuals is defined by equation $x(t,0) = \int\limits_0^L r(l,E(t,l))x^\beta(t,l)dl + p_0$. with some $\beta \in (0,1)$, birth rate $r \geq 0$ and industrial population renewal $p_0 \geq 0$. We show that under natural assumption there exists a measurable exploitation intensity which provides the maximum profit for the stationary mode exploitation for various objective functions.

Friday, May 15

37. Plenary — 08:45 – 09:35

***Chair*: Session Chair** — **Festsaal**

The minimum time function: old and new

Piermarco Cannarsa (1),

(1) University of Rome “Tor Vergata”, Rome, Italy

The classical time optimal control problem consists of steering a given control system to a prescribed target in minimum time. Optimal trajectories are therefore sensitive to the controllability properties of the dynamics as well as the geometrical structure of the target. This talk analyzes the regularity of the minimum time function and its dependance on the above two factors, and explains how sensitivity analysis can be used to derive to optimality conditions.

38. Optimal control in finite and infinite dimension and application to economics 3 — 10:00 – 11:15

***Chair*: S. Faggian, F. Gozzi** — **PrechtIsaal I**

Time-varying consumption tax, productive government spending, and aggregate instability

Mauro Bambi (1), Alain Venditti (2),

(1) University of York, York, U.K. (2) GREQAM, Marseille, France

In this paper we study the dynamics of an economy with productive government spending under the assumption that the government balances its budget by levying endogenous non-linear consumption taxes. For standard specification of the utility function and production function, we prove that under counter-cyclical consumption taxes, while there exists one unique intertemporal equilibrium characterized by a balanced growth path, sunspot equilibria based on self-fulfilling expectations occur through some form of global indeterminacy.

Durable good monopoly with network effects and its implications

Sebastian Aniţa (1), Stefan Behringer (2), Thorsten Upmann (3),

(1) Faculty of Mathematics, Alexandru Ioan Cuza University of Iaşi (2) Mercator School of Management, Universität Duisburg-Essen (3) Faculty of Business Administration and Economics, Universität Bielefeld

In this paper we investigate the validity of the Coase Conjecture for durable goods monopolies in the presence of network effects. The model allows for heterogenous consumer preferences that are dynamic on some product space. It is shown that the Coase Conjcture dramatically fails in this setting and regulatory implications for internet monopolies are derived.

The Coase Conjecture (Coase, 1972) implies that as the period over which a durable goods monopolist is able to make binding commitments decreases, the monopolist's ability to exert market power decreases. In the limit market power fully vanishes and the competitive outcome results. Intuitively the monopoly at any period is competing with an infinite series of copies of itself. This finding has important implications for welfare evaluations and thus regulatory concern in durable goods markets. Also, if the sale of a durable product creates a second- hand market that is not controlled by the monopolist (i.e. if rental is not feasible) then the monopolist is led to producing goods that are less durable than those that would result under competition, thus raising another regulatory concern. Coase's intuition has been rationalized formally using mathematical dynamic modelling only a decade later. Bulow (1982) and Stokey (1981) model the underlying rational expectations explicitly. Thépot (1998) shows that these may alternatively arise from the dynamic optimality conditions directly.

Utility maximization with state constraints and non-concave technology

Fausto Gozzi (1), Paolo Acquistapace (2), Francesco Bartaloni (2), Davide Fiaschi (3),

(1) Luiss University, Roma, Italy (2) Dipartimento di Matematica, Università di Pisa, Italy
(3) Dipartimento di Economia, Università di Pisa, Italy

We consider an optimal control problem arising in the context of economic theory of growth, on the lines of the works by Skiba (1978) and Askenazy - Le Van (1999). The economic framework of the model is intertemporal infinite horizon utility maximization. The dynamics involves a state variable representing total endowment of the social planner or average capital of the representative dynasty. From the mathematical viewpoint, the main features of the model are the following: (i) the dynamics is an increasing, unbounded and not globally concave function of the state; (ii) the state variable is subject to a static constraint; (iii) the admissible controls are merely locally integrable in the right half-line. Such assumptions seem to be weaker than those appearing in most of the existing literature. The results are the following.

- We give a direct proof of the existence of an optimal control for any initial value of the state variable and we carry on a qualitative study of the value function.
- Using dynamic programming methods, we show that the value function is a continuous viscosity solution of the associated Hamilton-Jacobi-Bellman equation.

- We prove differentiability properties of the value functions.
- We exploit the above results to study the behavior of the optimal trajectories improving the information on such trajectories given in the above quoted papers.

39. Mean-field models and control of multi-agent systems 1 — 10:00 – 11:15

***Chair*: G. Albi, M. Fornasier** — **PrechtIsaal II**

Collision avoidance in pedestrian dynamics

Adriano Festa (1), Marie-Therese Wolfram (1),

(1) RICAM, Austrian Academy of Sciences (ÖAW), Linz, Austria

Experiments indicate that one of the main forces in pedestrian dynamics is collision avoidance. In other words individuals actively anticipate the future to predict a possible collision time and adjust their velocity accordingly, see [2]. At the same time they show no intention to change their direction when walking close to each other in the same direction. This collision avoidance behaviour plays a major role in pedestrian dynamics, which was already illustrated in numerical simulations. The simulations show that already a simple dynamical systems model, with forces depending on the estimated hitting time reproduce complex dynamics such as velocity alignment. In this talk we propose a different model, motivated from optimal control theory, which is based on a simple two-player pursuer evader game. We shall use Bellman's approach [1] to study the embedded multi-player game for collision avoidance and discuss related theoretical as well as numerical aspects.

[1] Bardi, Martino, and Italo Capuzzo-Dolcetta. *Optimal control and viscosity solutions of Hamilton-Jacobi-Bellman equations*. Springer Science & Business Media, 2008.

[2] Karamouzas, Ioannis, Skinner, Brian and Guy Stephe. *Universal power law governing pedestrian interactions*. Physical review letters, 113(23) (2014). 238701.

On the interpretation of the Master equation

Alain Bensoussan (1,2),

(1) International Center for Decision and Risk Analysis, UT Dallas, US (2) Systems Engineering and Engineering Management, City University of Hong Kong

Since its introduction by P.L. Lions in his lectures and seminars at the College de France [5, 6], see also the very helpful notes of Cardialaguet on Lions' lectures [3], the Master equation has attracted a lot of interest, and various points of view have been expressed, see [1, 2, 3]. There are several ways to introduce this type of equation. It involves an argument which is a probability measure, and P.L. Lions has proposed the idea of working with the Hilbert space of random variables which are square integrable. So writing the equation is an issue. Another issue is its

origin. We discuss in this paper these various aspects, and for the modeling rely heavily on a seminar at College de France delivered by P.L. Lions on November 14, 2014.

[1] Bensoussan A., Frehse J., Yam P. The Master Equation In Mean Field Theory ,*Journal de Mathmatiques Pures et Appliques* , Paris, 2014.

[2] Buckdahn R., Li J., Peng S.G., Rainer C., Mean Field stochastic differential equations and associated PDEs *preprint*. 2014.

[3] Cardaliaguet P., Notes on P.L. Lions' lectures at the College de France, *Working Paper*, 2014.

[4] Carmona R., Delarue F., The Master Equation for Large Population Equilibriums, *arxiv*, 2014.

[5] Lions P.L., Lectures at College de France, *http://www.college-de-france.fr*, 2014.

[6] Lions P.L., Seminar at College de France, 2014.

Nonlocal interaction equations with many species.

Marco Di Francesco (1), Simone Fagioli (1),

(1) University of L'Aquila, Italy

I will present a series of results in collaboration with S. Fagioli (L'Aquila) in [1] and [2] dealing with a systematic theory for nonlocal interaction equations with two species, arising e. g. from population biology and social sciences. We first consider systems with symmetric cross interaction and prove that an existence and uniqueness theory can be performed in the context of the Wasserstein gradient flow theory developed by Ambrosio, Gigli, and Savaré. For non symmetric systems with mildly singular interaction, we present an existence result based on an implicit-explicit version of the Jordan-Kinderlehrer-Otto scheme. We prove finite-time blow up and segregation results by following an atomization strategy. We then consider the case of predator-prey interaction, and explore the stability of non trivial particle steady states. Finally, we propose a result on the existence of segregated steady states for a model with degenerate cross-diffusion.

[1] M. Di Francesco, and S. Fagioli, Measure solutions for nonlocal interaction PDEs with two species. *Nonlinearity*, 26 (2013), 2777-2808.

[2] M. Di Francesco and S. Fagioli, Steady states for a two species system of nonlocal interaction PDEs of predator-prey type. *Submitted preprint*

40. Growing economies and the environment — 10:00 – 11:15

***Chair*: M.C. Camacho, C. Saglam** — **Festsaal**

Tourism and environmental quality

Bazoumana Ouattara (1), Agustin Perez-Barahona (2), Eric Strobl (3),

(1) University of Manchester Manchester, UK (2) INRA and Ecole Polytechnique, France (3) Ecole Polytechnique, France

The United Nations Environment Programme points out the interaction between tourism and environmental quality. In particular, Christ *et al.* (2003) draw attention to the raising number of tourists seeking for destinations in environmental hotspots. In the same direction, Font (2000) observes that environmental protection projects encourage tourist activities. Moreover, it is also recognized the negative environmental impact of tourism (Green *et al.*, 1990; Green and Hunter, 1992; Gossling *et al.*, 2002; and Brei *et al.*, 2011). This double causality between tourism and environmental quality is, indeed, very important for the Caribbean. The rich environmental quality of the Caribbean makes it one of the most appealing destinations for tourism. However, the environmental deterioration due to the development of this sector (for instance, the effect of accommodation facilities and infrastructures on water quality, biodiversity, *etc.*) negatively affects the growth prospects of these countries. Our paper investigates the dynamical interaction between tourism and environmental quality. We empirically quantify this double causality by means of considering data of tourists' arrival and hotels, and bathing quality as proxy of environmental quality. Finally, we analytically describe the dynamical properties of the problem, pointing out the role of ecotourism and the existence of imbalance effects.

Transboundary pollution abatement: The impact of unilateral commitment in differential games

Luisito Bertinelli (1), Luca Marchiori (2), Amer Tabakovic (1), Benteng Zou (1),

(1) CREA, University of Luxembourg, Luxembourg (2) Central Bank of Luxembourg, Luxembourg

The present study explores the strategic interactions of countries setting pollution abatement policies in a dynamic two-player game. To reach a common target of environmental quality, countries can choose to commit to a stream of pollution abatement right from the beginning of the game or decide upon abatement at each moment in time. The main nov- elty of this paper resides in the introduction of heterogeneous strategies, where only one country commits to a specific abatement policy. And that is what we can observe cur- rently among large pollution nations, in contrast to homogenous strategies, where both or no country commits to a predefined policy. We find that accumulated pollution can be lower under heterogeneous strategies than under homogenous strategies depending on the target. A stringent environmental quality target will induce the committed player to produce an abatement effort that more than compensates the free-riding attitude of the non-committed player.

The consequences of a one-sided externality in a dynamic, two-agent framework

Georg Müller-Fürstenberger (1), Ingmar Schumacher (2,3),

(1) Deparment of Economics, Trier University, Germany (2) IPAG Business School, Paris, France (3) Department of Economics, Ecole Polytechnique Paris, France

We discuss a dynamic model where all agents contribute to a global externality, but only those in a specific region suffer from it. We model this in a dynamic setting via a two agent, non-cooperative overlapping generations model and analyze the consequences for economic growth and intertemporal choices. We find that multiple steady states may result from this asymmetry. In particular, if the agent who is affected by the externality has to spent a large share of his income to offset it, then he may be stuck in an environmental poverty trap. We provide conditions for the existence of, and local convergence to, the equilibria, as well as a condition for the global convergence to the poverty trap.

While, in addition to maintenance expenditures, externalities tend to be addressed via studying taxes, investment in R&D or alike, we focus on capital market integration. Specifically, agents in the affected region can open up their capital market to enable capital inflows. We investigate whether an open capital market improves or worsens their welfare. While we do find that capital market integration eliminates the environmental poverty trap, we show that capital market integration is not always in both agents' interest. In particular, we provide conditions under which the agents prefer autarkic or integrated capital markets.

The authors are grateful to Gianluigi Vernasca and participants at the EAERE conference in Rome, 2011. This article previously circulated under the title "Is Capital Market Integration a Remedy for the Environmental Poverty Trap?"

41. Stochastic control and applications 2 — 10:00 – 11:15

***Chair*: M. Falcone, H. Zidani** — **Böcklsaal**

Ergodicity of optimal diffusion processes on $\mathbb{R}^d$

Marco Cirant (1),

(1) Università di Milano, Italy

In this talk we consider ergodic control problems where both the state and the control space are unbounded. The focus is to obtain some general conditions on the fixed drift, the cost function, and the Lagrangian function that are sufficient for synthesizing an optimal control of feedback type. Moreover, we look for an optimal process which is ergodic, namely it approaches its invariant measure in the long-time regime.

In order to obtain such conditions, and to overcome the lack of compactness of the state and control spaces, we propose an approach that combines the Lyapunov method and the approximation of the problem on bounded sets. In particular, we study the limit of solutions of the associated HJB equation on balls with increasing radius, with Neumann conditions at the boundary.

We first develop a general framework, and then study particular cases which show how Lyapunov functions can be constructed from the solutions of the approximating problems.

Numerical Dynamic Programming techniques for stochastic hybrid systems

Roberto Ferretti (1),

(1) Dipartimento di Matematica e Fisica, Università Roma Tre, Roma, Italy

The notion of hybrid systems has been recently proposed in order to provide a unified framework for control systems subject to both continuous and discrete control actions. In the case of stochastic hybrid control, a sound theoretical framework for the application of dynamic programming techniques is given in [1].
Following a previous work [2] on the application of monotone numerical schemes to the quasi-variational inequality arising in the deterministic hybrid case, we investigate the generalization of these ideas to a stochastic hybrid optimal control problem, discussing construction and convergence of monotone schemes, and presenting numerical examples to validate the theoretical framework.

[1] A. Bensoussan and J.L. Menaldi. Stochastic Hybrid Control. *J. Math. Anal. and Appl.*, 249(1): 261–288, 2000.

[2] R. Ferretti and H. Zidani. Monotone numerical schemes and feedback construction for hybrid systems. *J. of Optim. Theory and Appl.*, DOI: 10.1007/s10957-014-0637-0.

Piecewise constant policy approximations to Hamilton-Jacobi-Bellman equations

Peter A. Forsyth (1), Christoph Reisinger (2),

(1) Cheriton School of Computer Science, University of Waterloo, Canada (2) Mathematical Institute, University of Oxford, United Kingdom

We establish a convergence analysis of discretization schemes for Hamilton-Jacobi-Bellman (HJB) equations with piecewise constant policy time-stepping. An advantageous feature of the proposed policy time-stepping method is that different linear approximation schemes, and indeed different meshes, can be used for the resulting sub-problems with different constant control parameters. Using the equivalence to a switching system and an adaptation of the usual arguments around consistency, stability and monotonicity, we show that if limited, potentially higher order interpolation is used for the mesh transfer, convergence is guaranteed. We provide numerical tests for the mean-variance optimal investment problem and the uncertain volatility option pricing model, and compare the results to published test cases.

42. Optimal control in finite and infinite dimension and application to economics 4

11:30 – 12:45

***Chair*: S. Faggian, F. Gozzi** **Prechtlsaal I**

A ricardian model of forestry

Giuseppe Freni (1), Silvia Faggian (2),

(1) Department of Business and Economics, University of Naples *Parthenope*, Naples, Italy
(2) Department of Economics, University *Ca' Foscari* Venice, Italy

This paper provides a continuous-time "Ricardian" model of forestry, where, in response to an increase in timber demand, forest cultivation is progressively intensified on the most fertile

lands and/or extended to less fertile qualities of lands. It is shown that, at a given level of the rate of interest, a set of “break through timber prices” gives the *order of fertility* (i.e., the order in which the different qualities of land are taken into cultivation) and that, for each land, there is a “threshold timber price” above which the prices of trees standing on the land are positive. Since for each land the “break through price” is higher than the “threshold price”, Ricardo is shown to be right that, given not all of what “is annually paid by a farmer to his landlord” can be considered rent, a higher demand for timber could simply raise those components of the compensation that are not rent.

On the maximum principle for impulsive optimal control problems with infinite time horizon

Anton O. Belyakov (1), Andrea Seidl (2), Gernot Tragler (2),

(1) Institute of Mechanics, Lomonosov Moscow State University, Moscow, Russia (2) ORCOS, Vienna University of Technology, Vienna, Austria

We consider optimality conditions for impulsive control problems where the time horizon is infinite. The maximum principle including transversality condition is proved for both finite and infinite values of objective functional using the concept of weakly overtaking optimality. The special case of a periodic optimal solution is considered within an illustrative example.

Necessary conditions for a weak minimum in optimal control problems with integral equations on a variable time interval

Andrei v. Dmitruk (1,2), Nikolai P. Osmolovskii (3,4),

(1) Russian Academy of Sciences, Central Economics and Mathematics Institute, Moscow, Russia (2) Lomonosov Moscow State University, Moscow, Russia. (3) University of Technology and Humanities in Radom, Poland (4) Moscow State University of Civil Engineering, Moscow, Russia

We study an optimal control problem with Volterra-type integral equation, considered on a nonfixed time interval, subject to endpoint constraints of equality and inequality type. We obtain first-order necessary optimality conditions for an extended weak minimum, the notion of which is a natural generalization of the notion of weak minimum with account of variations of the time. The conditions obtained generalize the Euler–Lagrange equation and transversality conditions for the Lagrange problem in the classical calculus of variations with ODEs. Their novelty, as compared with those for problems on a fixed time interval is that the costate equation and transversality condition with respect to time variable involve nonstandard terms that are absent in problems with ODEs. The proof is based on reducing the problem to a problem on a fixed time interval and then using the general Lagrange multipliers rule. However, in contrast to problems with ODEs, this reducing leads to an integral equation which is not of the standard Volterra-type, and this is the cause of appearance of the new additional terms.

43. Mean-field models and control of multi-agent systems 2 — 11:30 – 12:45

***Chair*: G. Albi, M. Fornasier** — **Prechtlsaal II**

Conservation laws with non-local flux in traffic flow modeling

Sebastien Blandin (1), Paola Goatin (2), Sheila Scialanga (2,3),

(1) IBM Research Collaboratory, Singapore (2) Inria Sophia Antipolis - Méditerranée, France (3) Università di Roma I - La Sapienza, Italy

Conservation laws with non-local flux have been recently introduced in traffic flow modeling to account for the reaction of drivers or pedestrians to the surrounding density of other individuals, see [1,3-6,8]. In this talk, we will consider the following mass conservation equation with non-local velocity

$$\partial_t \rho(t,x) + \partial_x \left(\rho(t,x) v\Big(\int_x^{x+\eta} \rho(t,y) w_\eta(y-x)\, dy \Big) \right) = 0\,, \qquad (1)$$

defined for $t \in \mathbb{R}^+$ and $x \in \mathbb{R}$, $\eta > 0$. The convolution kernel $w_\eta \in \mathcal{C}^1([0,\eta];\mathbb{R}^+)$ is assumed to be a non-increasing function such that $\int_0^\eta w_\eta(x)dx = 1$.

We show how to prove the well-posedness of entropy weak solutions of (1) by providing accurate L^∞, BV and L^1 estimates for the sequence of approximate solutions constructed by an adapted Lax-Friedrichs scheme, see [2,7].

[1] P. Amorim, R.M. Colombo, A. Teixeira. A numerical approach to scalar nonlocal conservation laws. *ESAIM Math. Model. Numer. Anal.*, 49(1), 19–37, 2015.

[2] S. Blandin, P. Goatin. Well-posedness of a conservation law with non-local flux arising in traffic flow modeling. Preprint, 2014.

[3] R.M. Colombo, M. Garavello, M. Lécureux-Mercier. A class of nonlocal models for pedestrian traffic. *Mathematical Models and Methods in Applied Sciences*, 22(04), 1150,023, 2012.

[4] R.M. Colombo, M. Herty, M. Mercier. Control of the continuity equation with a non local flow. *ESAIM Control Optim. Calc. Var.*, 17(2), 353–379, 2011.

[5] R.M. Colombo, M. Lécureux-Mercier. Nonlocal crowd dynamics models for several populations. *Acta Math. Sci. Ser. B Engl. Ed.*, 32(1), 177–196, 2012.

[6] G. Crippa, M. Lécureux-Mercier. Existence and uniqueness of measure solutions for a system of continuity equations with non-local flow. *Nonlinear Differential Equations and Applications NoDEA*, 20(3), 523-537, 2013.

[7] S. Scialanga. Traffic flow models with non-local velocity. Tesi di laurea, Università di Roma "La Sapienza", 2015.

[8] A. Sopasakis, M.A. Katsoulakis. Stochastic modeling and simulation of traffic flow: asymmetric single exclusion process with Arrhenius look-ahead dynamics. *SIAM J. Appl. Math.*, 66(3), 921–944 (electronic), 2006.

Meanfield games and model predictive control

Michael Herty (1),

(1) RWTH Aachen University, Aachen, Germany

Mean-Field Games are games with a continuum of players that incorporate the time-dimension through a control-theoretic approach. Recently, simpler approaches relying on the Best Reply Strategy have been proposed. They assume that the agents navigate their strategies towards their goal by taking the direction of steepest descent of their cost function (i.e. the opposite of the utility function). In this talk, we explore the link between Mean-Field Games, Riccati based control and the Best Reply Strategy approach. This is done by introducing a Model Predictive Control framework, which consists of setting the Mean-Field Game over a short time interval which recedes as time moves on. We show that the Model Predictive Control offers a compromise between a possibly unrealistic Mean-Field Game approach and the sub-optimal Best Reply Strategy.

A smooth and localized version of the Hughes model for pedestrian flow

Stephan Martin (1, 2), José Antonio Carrillo (1), Marie-Therese Wolfram (3),

(1) Department of Mathematics, Imperial College London, UK (2) Department of Mathematics, RWTH Aachen University, Germany (2) Radon Institute for Computational and Applied Mathematics, Linz, Austria

In the classical model of Hughes for crowd motion [1] pedestrians seek to minimize their travel time to a-priori known destinations/exits, but try to avoid regions of high density. One of the basic assumptions is that the overall density of the crowd is known to every agent at every point in time. We present results on a modification that includes localizing effects such as limited vision to a Hughes-type equation. The basic mechanism permits agents to perceive information on the current crowd density only in a local neighborhood, while taking assumptions on the density outside that region. Furthermore, we suggest a smoothing operator for the velocity field and a local interaction based on conviction. The model is presented on both a microscopic and macroscopic perspective, and we illustrate the resulting dynamics in an evocation scenario. Surprisingly, it will turn out that evacuation times can even improve in some situations. The question we investigate is hence complementary to mean-field game approaches to crowd dynamics, where pedestrians anticipate future crowd states and therefore are *more* capable than in the original Hughes' model.

[1] R.L.Hughes. A continuum theory for the flow of pedestrians. *Transportation Research Part B: Methodological, 36(6). (2002), 507-535.*

44. Optimal management in complex environments 1 — 11:30 – 12:45

***Chair*: F. Wagener** — **Festsaal**

Unconditional aid and green growth

Moutaz Altaghlibi (1,2,3),

(1) Universite Paris 1 Pantheon-Sorbonne, Paris, France (2) University of Amsterdam, Amsterdam, Netherlands (3) European Doctorate in Economics Erasmus Mundus (EDEEM)

Climate change is one of the most important problem nowadays due to its adverse effects on all countries regardless of their development level, this is why green investments are no longer just a luxury, but can be considered as a social and legal responsibility. Countries at a low level of development have a challenge to achieve their economic growth and preserve the environment at the same time, and this is can be done by the having aid from developed countries that can help these countries invest in green rather than dirty investments. We study a differential open-loop stackelberg game between two countries; developed country (leader) and developing country (follower). At any point of time the leader assumed to have a choice between two strategies; consuming or giving unconditional aid to the follower, while the follower chooses one of three strategies; investing in green or dirty investments, or consuming. We solve numerically for the optimal paths of both players and study how the choice of each player will change with different settings of the game. The steady state analysis for the follower's problem shows that aid would have an effect in decreasing the sustainable levels of dirty capital, dirty investments, and the stock of pollution, and increasing the sustainable level of consumption, while having no effect on the sustainable levels of green capital and green investments.

Heterogeneous expectations and climate catastrophes

Tatiana Kiseleva (1),

(1) Netherlands Bureau for Economic Policy Analysis, The Hague, The Netherlands

We study how heterogeneous beliefs about the causes and extent of global warming affect local mitigation and adaptation strategies and therefore global climate dynamics. Local policies are determined by expectations of policy makers about future climate. There are three types of expectations: strong skeptic, weak skeptic and rational. Strong skeptics deny anthropogenic climate change and possibility of a climate catastrophe. Weak skeptics believe that industrial CO_2 emissions cause global warming, but deny catastrophic climate change. Rational policy makers consider both: human influence on climate and possible catastrophic shifts. Aggregate behavior of policy makers determines the total emission level which influences global climate dynamics. The paper argues that rational expectations are not crucial for preventing the catastrophe.

Catastrophe and cooperation

Pim Heijnen (1), Lammertjan Dam (1),

(1) University of Groningen, The Netherlands

In this paper, we develop a parsimonious model of international environmental agreements. We argue that the three key issues that need to be addressed are that climate change is catastrophic, that countries are sovereign (and hence there are participation constraints in designing international environmental agreement) and that countries differ in their exposure to climate change. Technically, this leads to a stochastic game with an absorbing state whose equilibrium structure is very different from the infinitely repeated games that are usually studied in the literature on environmental agreements. In particular there is no "folk theorem" that guarantees that the social optimum can be sustained in a Nash-equilibrium with the aid of trigger strategies as long as players are sufficiently patient. However, in most circumstances, it is feasible to implement an abatement scheme with a level of aggregate abatement that is close to the social optimum.

45. Distributed control in economics and management — 11:30 – 12:45

***Chair*: Session Chair** — **Böcklsaal**

An optimal control problem related to an invasive species model

Narcisa C. Apreutesei (1),

(1) Department of Mathematics and Informatics, "Gheorghe Asachi" Technical University of Iasi, Romania

Consider a reaction-diffusion system that models the dynamics of a population of invasive species, where the normal females and males are taken into account. We introduce a "supermale" and a "feminized supermale" into the population, with YY chromosomes. They are genetically modified organisms. Mating with them produces only males and supermales. The long time behavior is that this will lead to less and less number of normal females in the population, ultimately leading to extinction. This is a way to eradicate a harmful invasive species. These "supermales" and "feminized supermales" are created by hormone treatments in a laboratory environment, then released into the population. The model has also applications in salmon industry to produce more male offspring. We regard the model as an optimal control problem. Suppose that the "feminized" mutant is released at some rate into the population and this rate is regarded as a control parameter. Our goal is to minimize the female population, maximize the male population, while minimizing the rate of introduction of the "feminized supermales". We prove the existence of an optimal solution and establish some optimality conditions for the above problem.

The influence of consumer recommendations on advertising strategies in a non-linear optimal goodwill model with market segmentation

Dominika Bogusz (1), Mariusz Górajski (1),

(1) Department of Econometrics, University of Łódź, Poland

We propose a new dynamic model of product goodwill where a product is sold in many market segments, and where the segments are indicated by the usage experience of consumers.

The main novelty of this model is that the product goodwill in a segment of new consumers depends not only on advertising effort, but also on consumer recommendations, for which we introduce a mathematical representation. The dynamics of product goodwill is described by a partial differential equation and is used in an optimal control model (see equation (1) in [1]). The control variable is the company's advertising efforts different in each segments. The aim of the control model is maximized company's profits. Using the maximum principle from [2], we numerically find the optimal advertising strategies and corresponding optimal goodwill paths. The sensitivity of these solutions is analysed. We identify two types of optimal advertising campaign: 'strengthening' and 'supportive'. They may assume different shapes and levels depending on the market segment and time. These experiments highlight the need for both researchers and managers to consider a segmented advertising companies and consumer recommendation in creating successful policy in enhancing product goodwill and building brand loyalty.

[1] D. Bogusz, M. Górajski. Optimal double control problem for a PDE model of goodwill dynamics. *arXiv:1411.0880 [math.OC]*, 2014.

[2] G. Feichtinger, G. Tragler, V. Veliov. Optimality conditions for age-structured control systems. *Journal of Mathematical Analysis and Applications* 288 (1), 47–68, 2003.

Optimal management and spatial patterns in a distributed shallow lake model

Hannes U. Uecker (1), Dieter Grass (2),

(1) Institut für Mathematik, Universität Oldenburg, D26111 Oldenburg, Germany (2) ORCOS, Vienna University of Technology, Vienna, Austria

We present a framework to numerically treat spatially distributed optimal control problems with an infinite time horizon. The basic idea is to consider the associated canonical system in two steps. First we perform a bifurcation analysis of the steady state canonical system using the continuation and bifurcation package pde2path, yielding a number of so called flat and patterned canonical steady states. In a second step we link pde2path to the two point boundary value problem solver `TOM` to study time dependent canonical system paths to steady states having the so called saddle point property. As an examples we consider a shallow lake model with diffusion, and optimal harvesting in a vegetation model.

46. Stability and numerical methods for optimal control 1 — 14:00 – 15:40

***Chair*: W. Alt, U. Felgenhauer** — **Prechtlsaal I**

Relaxed optimal control of systems defined by integral equations

John Coletsos (1), Basil Kokkinis (1),

(1) National Technical University, Athens, Greece

We consider an optimal control problem for systems defined by nonlinear Volterra integral equations, with control constraints, final equality state constraints, and pointwise inequality state

constraints. Without any convexity assumptions on the data, it is well known that these problems have no classical solutions in general. Therefore the optimal control problem is reformulated into its relaxed form. We then prove the existence of an optimal relaxed control and derive necessary conditions for optimality in the form of a relaxed minimum principle of Pontryagin type with transversality conditions. Moreover, and under additional differentiability assumptions, we apply a mixed descent-penalty method which generates sequences of relaxed controls converging to extremal controls, i.e. controls which satisfy the relaxed necessary conditions for optimality. A numerical example is also given.

Thermostatic approximation of optimal control problems on multi-domains

Fabio Bagagiolo (1), Rosario Maggistro (1),

(1) Department of Mathematics, University of Trento, Italy

We study an optimal control problem with two controlled different dynamics in two half-spaces, in the framework of HJB equations. We introduce an approximation by the use of switching rules of the delayed-relay type (see for example [1] for this kind of switching optimal control problems), and we study the passage to the limit when the parameter of the approximation (i.e. the switching-thresholds distance) goes to zero. We then compare our result with other ones from the recent literature (in particular with [2], see also [3]). Finally, we briefly sketch a one-dimensional threefold junction problem.

[1] F. Bagagiolo, K. Danieli. Infinite horizon optimal control problems with multiple thermostatic hybrid dynamics. *Nonlinear Analysis: Hybrid Systems*, 6:824–838, 2012.

[2] G. Barles, A. Briani, E. Chasseigne. A Bellman approach for two-domains optimal control problems in $\mathbf{R}^N$. *ESAIM Control Optim. Calc. Var.*, 19:710–739, 2013.

[3] Z. Rao, A. Siconolfi, H. Zidani. Transmission conditions on interfaces for Hamilton-Jacobi-Bellman equations. *J. Differential Equations*, 257:3978–4014, 2014.

Euler type discretization for bang-singular control

Ursula Felgenhauer (1),

(1) Brandenburg University of Technology Cottbus – Senftenberg, Germany

Variational inequalities and Robinson's theory of strongly regular generalized equations are appropriate tools for analyzing several classes of optimal control problems. For control-affine systems and bang-singular control behavior, after some adaptation this approach yields existence and Lipschitz stability results for extremals of the problem under (known) second-order and structural stability conditions.

The implicit discretization will start from a variational inequality formulation of first-order necessary optimality conditions. However, the weakness of coercivity conditions and the need of utilizing Goh's transformation cause specific difficulties. Restricting the problem class to so-called semilinear control systems and using additional growth restrictions for the state equation,

one can prove a first convergence result of order one for Euler type discretization. An example illustrates the method and some regularization.

Convergence analysis for discretized control-state constrained optimal control problems with controls of bounded variation

Matthias Gerdts (1),

(1) Institute of Mathematics and Applied Computing, University of the Federal Armed Forces at Munich, Neubiberg, Germany

We study convergence properties of discretized optimal control problems with ordinary differential equations and mixed control-state constraints. Under suitable consistency and stability assumptions a convergence rate of order $1/p$ of the discretized control to the continuous control is established in the L^p-norm. Throughout it is assumed that the optimal control is of bounded variation. The convergence proof exploits the reformulation of first order necessary optimality conditions as nonsmooth equations.

47. Infinite horizon optimal control and mathematical economics 3 14:00 – 15:40

***Chair*: A. Zaslavski** **PrechtlsaalII**

The management of fragile resources: a long term perspective

Yacov Tsur (1), Amos Zemel (2),

(1) Department of Agricultural Economics and Management, The Hebrew University of Jerusalem, Israel (2) Department of Solar Energy and Environmental Physics, Ben Gurion University of the Negev, Israel

Excessive exploitation diminishes the capacity of natural resources to withstand environmental stress, increasing their vulnerability to extreme conditions that may trigger abrupt changes. The onset of such events depends on the coincidence of random environmental conditions and the resource state (determining its resilience). Examples include species extinction, ecosystem collapse, disease outburst and climate change induced calamities. The policy response to the catastrophic threat is measured in terms of its effect on the long-term behavior of the resource state. To that end, the L-methodology, developed originally to study autonomous systems, is extended to non-autonomous problems involving catastrophic threats.

Cooperation in noisy repeated games with random matching

Chaowen Yu (1),

(1) Keio University, Tokyo, Japan

The purpose of this paper is to investigate whether the two players in a long-run relationship can maintain cooperation when neiher of them exactly knows the detail of the underlying game

under the random matching setting. We show that when the discount factor is close to unity and the population is large enough, any efficient outcomes can be attained as sequential equilibrium outcomes. As an application, we derive a version of the folk theorem for noisy prisoner's dilemma games in the large-population random matching setting.

Structure of solutions of variational problems with extended-valued integrands in the regions close to the endpoints

Alexander J. Zaslavski (1),

(1) Department of Mathematics, The Technion – Israel Institute of Technology, Haifa, Israel

We study the structure of approximate solutions of autonomous variational problems with a lower semicontinuous extended-valued integrand. In our recent research we showed that approximate solutions are determined mainly by the integrand, and are essentially independent of the choice of time interval and data, except in regions close to the endpoints of the time interval. In this talk we discuss the structure of approximate solutions in regions close to the endpoints of the time intervals.

On limiting gradients of value function for infinite horizon problem

Dmitry V. Khlopin (1, 2),

(1) Krasovskii Institute of Mathematics and Mechanics, Yekaterinburg, Russia (2) Ural Federal University, Yekaterinburg, Russia

We investigate necessary conditions of optimality for the infinite horizon problem with free right end. The optimality is understood in the sense of weakly uniformly overtaking optimal control. No previous knowledge of the asymptotic behaviour of trajectories or adjoint variables is necessary. We obtain the necessary transversality condition at infinity; this condition may be expressed both with the aid of an Aseev–Kryazhimskii-type formulae for co-state arcs, and through limiting gradients of payoff function at $t = \infty$ [1].

An important case is when the value function V exists. As can be proved that in this case the Lipschitz continuity of V implies the normal form version of the Pontryagin maximum principle; i.e. the control problem admits normal co-state arc ψ^*. Moreover, $-\psi^*(0)$ becomes a limiting gradient of V.

[1] D.V. Khlopin Necessity of limiting co-state arc in Bolza-type infinite horizon problem. *Optimization*, published online: 20 Oct 2014, arXiv:1407.0498

48. Optimal management in complex environments 2 — 14:00 – 15:40

***Chair*: F. Wagener** — **Festsaal**

Sound auction specification and implementation

Colin Rowat (1), Marco B. Caminati (2), Manfred Kerber (2), Christoph Lange (3),

(1) Department of Economics, University of Birmingham (2) School of Computer Science, University of Birmingham (3) Fraunhofer IAIS and University of Bonn, Germany

We introduce 'formal methods' of mechanized reasoning from computer science to address two problems in auction design and practice: is a given auction design soundly specified, possessing its intended properties; and, is the design faithfully implemented when actually run? Failure on either front can be hugely costly in large auctions. In the familiar setting of the combinatorial Vickrey auction, we use a single mechanized reasoner, Isabelle, to first ensure that the auction has a set of desired properties (e.g. allocating all items at non-negative prices), and to then generate verified executable code directly from the specified design. Having established the expected results in a known context, we intend next to use formal methods to verify new auction designs.

Path dependence in a dynamic R&D model with an endogenous hazard rate

Herbert Dawid (1,2), Michel Y. Keoula (1), Michael Kopel (3), Peter M. Kort (4,5),

(1) Department of Business Administration and Economics, Bielefeld University (2) Center for Mathematical Economics, Bielefeld University (3) Department of Organization and Economics of Institutions, University of Graz (4) Department of Econometrics and Operations Research & CentER, Tilburg University, The Netherlands (5) Department of Economics, University of Antwerp, Belgium

We study in a dynamic framework how product innovation activities of a firm are influenced by its production capacity investments for an established product and vice versa. The firm initially has capacity to sell an established product, and it also has the option to undertake an R&D project, which upon completion allows the firm to introduce a new vertically and horizontally differentiated product to the market, thereby extending its product range. The breakthrough probability of detecting the new product depends on both the value of the firm's R&D stock and its current R&D investment. It is shown that the initial production capacity for the established product influences the intensity of R&D activities of the firm. In particular, there are constellations such that for large initial production capacity for the established product the firm never invests in R&D and the new product is never introduced. For small initial capacity the firm keeps investing in R&D implying that eventually the new product is always introduced. Finally, for an intermediate range of initial capacity levels the firm initially invests in product R&D, but then reduces these investments to zero. In this scenario the new product is introduced with a positive probability, which is however substantially smaller than 1. From a technical perspective this analysis gives the example of a new type of Skiba threshold phenomenon in the framework of a multi-mode optimization model.

A patent portfolio race with knowledge accumulation

Hsin-Mien Wang (1,2), Florian Wagener (1),

(1) EDEEM, Bielefeld University, Germany (2) CeNDEF, University of Amsterdam, Netherlands

This article models a patent portfolio race where firms contribute to a public knowledge stock, which enables them to innovate. Firms make R&D efforts and invest in labor to assimilate public knowledge so that they can innovate and build up a patent portfolio by patenting their innovations over time. We introduce a differential game involving a single public knowledge stock in which firms invest strategically to innovate. The auxiliary system introduced by Dockner and Wagener [1] is applied to characterize Markov-perfect Nash equilibria. Numerical methods allow us to find the optimal vector fields in the system and draw bifurcation graphs with different parameters. We find that firms innovate eventually if the total innovation benefit is high or time preference is low. Compared with high innovation benefit and low time preference, a strong patent protection system has less tendency to encourage innovations; nevertheless, it may reduce expected innovation time. To encourage innovation, a policy maker can subsidize public knowledge so that there is a sufficient initial stock of fundamental knowledge for firms to innovate in the long run.

[1] E.J. Dockner, F.O.O. Wagener. Markov-Perfect Nash Equilibria in Models With a Single Capital Stock. *CeNDEF Working Papers* 06-07, Universiteit van Amsterdam, Center for Nonlinear Dynamics in Economics and Finance, 2006.

Evolutionary environmental games

Paolo Zeppini (1),

(1) University of Bath, UK

We propose a model for the strategic interaction of a large number of players in environmental public goods such as lakes, river basins, woodlands, or crop plantations. We specifically consider complex ecological system with a non-linear response to human pollutant activities. The starting point of analysis is the economy of shallow-lakes. Instead of the traditional game theoretic setting of interaction between two or few perfectly rational strategic players, we adopt an evolutionary approach with a large population of myopic players. Each player represents an economic agent with an incentive to exploit the environmental public good. In the case of a lake, agents can be farmers and firms, but also families and tourists. In all cases the public good problem builds on the trade-off between the quality of the "environmental" services provided by the ecological system. The nature of the 2×2 stage interaction game changes with the state of the ecosystem. We are able to classify different conditions for stable clean regimes, stable polluted regimes, stable regimes with co-existence of cooperators and free-raiders, weakly stable equilibria, and dynamic regimes. In particular dynamic regimes present a rich variety of long-run attractor, including chaotic dynamics.

49. Harvesting 2 — 14:00 – 15:40

***Chair*: S. Behringer, T. Upmann** — **Böcklsaal**

On the uniqueness of the solution to an optimal control problem subject to a parabolic differential equation. The case of land use

Carmen Camacho (1), Agustin Perez-Barahona (2),

(1) CNRS and Universite Paris 1 Pantheon-Sorbonne, Paris, France (2) INRA and Ecole Polytechnique, Paris, France

In the recently revisited literature of Spatial Economics, a decision maker chooses the optimal levels of control variables for a given spatial region and time span, like consumption or land devoted to agriculture, taking into account how some state variables flow over space. It is an optimal control problem where the state equations are parabolic partial differential equations. Although a set of necessary conditions can be obtained, the existence of a unique solution has not been proven. We say then that the problem is ill-posed in the sense of Hadamard, that is, one cannot ensure existence nor uniqueness of the solution. In this paper and under fairly general assumptions, we prove the unicity of the solution when the time horizon is finite, solving the ill-posedness problem. We apply our results to the recent paper Camacho and Perez-Barahona (2015) on land use and pollution.

An impulse control approach to irreversible investment with fixed costs

Salvatore Federico (1), Mauro Rosestolato (2), Elisa Tacconi (3),

(1) University of Milano, Italy (2) LUISS University, Rome, Italy (3) Bocconi University, Milano, Italy

We consider an impulse control problem associated to a firm producing goods (electricity, oil,...), whose capacity can be increased by investing capital at random times $\{\tau_n\}_{n\in\mathbb{N}}$. The investments are irreversible and with fixed costs. We prove that the value function is the unique viscosity solution of the associated quasivariational inequality, we provide the analytic solution of the QVI, and we prove the verification theorem.

Exploitation of a renewable resource situated at discrete patches

Stefan Behringer (1), Peter M. Kort (2), Thorsten Upmann (3),

(1) University of Duisburg-Essen, Mercator School of Management, Lotharstraße 65, 47057 Duisburg, Germany (2) Tilburg University, Tilburg School of Economics and Management, PO Box 90153, 5000 LE Tilburg, The Netherlands (3) Bielefeld University, Faculty of Business Administration and Economics, Universitätsstraße 25, 33615 Bielefeld, Germany, CESifo, München, Germany

We contribute to the analysis of optimal harvesting of a renewable natural resource. While in the usual approach the resource is located at a single point, recent contributions challenge this view and assume that the resource is continuously distributed in space (see, e.g. [1] and [2]). In the continuous approach, the agent is allowed to do harvesting in an *en passant* manner, that is, the agent gathers the resource while simply passing it. In this paper, we dispense with both views: we assume that the resource is localised at discrete patches, and that at any those patches the process of harvesting takes time. This implies that upon arrival at a patch, the agent has to reduce his speed, to stop and to stay there for some time period, as these are prerequisites for any harvesting process. While these technical features enhance the realism in natural

resource modelling, they substantially affect the optimal path of movement and harvesting. Finally, a numerical analysis demonstrates that this modification may affect the optimal path quite significantly.

[1] Behringer, S. and T. Upmann (2014). Optimal Harvesting of a Spatial Renewable Resource. *Journal of Economic Dynamics and Control*, 42 (May), 105-120.

[2] Belyakov, A. O., A. Davydov, and V. M. Veliov (2013). Optimal Cyclical Exploitation of Renewable Resources. *ORCOS Research Report 2013-07*, Vienna University of Technology. http://orcos.tuwien.ac.at/research/research reports/.

Robust spatial harvesting

Athanasios N. Yannacopoulos (1), Anastasios Xepapadeas (2),

(1) Department of Statistics, Athens University of Economics and Business, Greece (2) Department of International and European Economic Studies, Athens University of Economics and Business, Greece

We consider models of harvesting a renewable resource in a spatial context where resource biomass moves within a spatial domain. Spatio-temporal model uncertainty could be present, as well as accumulation of pollution that may reduce the biomass growth. By employing methods and techniques of spatiotemporal dynamic analysis and robust control theory, we develop optimal harvesting rules and potentially environmental policy. We also elaborate on the effects of uncertainty on the qualitative aspects of the model. In particular possible deviations from the targets, arising from the interaction between spatial variability and uncertainty, are analyzed (hot spots) and their connection with pattern formation is discussed.

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50. Optimal control in finite and infinite dimension and application to economics 5

16:00 – 17:40

***Chair*: S. Faggian, F. Gozzi** **Prechtlsaal I**

Viscosity solutions of Hamilton-Jacobi equations in metric spaces.

Wilfrid Gangbo (1), Andrzej Swiech (1),

(1) School of Mathematics, Georgia Institute of Technology, Atlanta, Georgia, USA

We will present a theory of metric viscosity solutions based on the notion of local slopes for a class of Hamilton-Jacobi equations in metric spaces. We will discuss the main techniques and

present comparison and existence results. In particular, for equations with convex Hamiltonians we will look at the associated variational problems, the Hopf-Lax formula, and the fact that the value function is a metric viscosity solution.

Reflected BSDEs and optimal control and stopping for infinite-dimensional systems

Federica Masiero (1), Marco Fuhrman (2), Gianmario Tessitore (1),

(1) Università di Milano Bicocca, Dipartimento di Matematica e Applicazioni, via Cozzi 55, 20125 Milano, Italy (2) Politecnico di Milano, Dipartimento di Matematica, piazza Leonardo da Vinci 32, 20133 Milano, Italy

In this talk we introduce the notion of mild supersolution for an obstacle problem in an infinite dimensional Hilbert space. The minimal supersolution of this problem is given in terms of a reflected BSDEs in an infinite dimensional Markovian framework. The results are applied to an optimal control and stopping problem. The talk is based on a joint work with M. Fuhrman and G. Tessitore.

Infinite dimensional methods for path-dependent equations

Giovanni A. Zanco (1), Franco Flandoli (1), Francesco Russo (2),

(1) Dipartimento di Matematica, Università di Pisa, Italy (2) Unité de Mathématiques appliquées, ENSTA-ParisTech, France

I will present recent results about infinite dimensional methods for path-dependent stochastic differential equations and partial differential equations.
These non-markovian equations, whose coefficients are allowed to depend on the whole trajectory of the solution up to the present time, are a powerful tool in modeling complex evolution systems with memory that appear in finance, engineering and biology.
The infinite dimensionale methods I will introduce are a counterpart of the functional calculus recently developed by Dupire, Cont and Fournié and provide a link with the theory of delay equations and with infinite dimensional stochastic analysis, allowing to study path-dependent equations using standard differential and topological structures. They can be used to obtain classical solutions to path-dependent Kolmogorov-type PDEs and Feynman-Kac representation formulas on the spaces of continuous paths and c dl g paths, and moreover to prove Ito-type formulas for functionals of paths of semimartingales and of window processes.
They also provide an insight on the role and the analytical structure of the so-called horizontal derivative, which is a key object in the study of path-dependent equations. Furthermore they are suitable to be generalized and extended in order to be applied to optimal control problems with delay.

Functional versus Banach space stochastic calculus, and strong-viscosity solutions to semilinear parabolic PDEs and path-dependent PDEs

Andrea Cosso (1), Francesco Russo (2),

(1) LPMA, Université Paris Diderot, Paris, France (2) ENSTA ParisTech, Paris, France

In the first part of the talk we revisit the basic theory of functional Itô calculus, using the formulation of calculus via regularization. This allows us to explore its relations with the corresponding Banach space stochastic calculus. Secondly, we introduce a viscosity type solution, called strong-viscosity solution to distinguish it from the classical one, with the following peculiarities: it is a purely analytic object; it can be easily adapted to more general equations than classical partial differential equations. We introduce the notion of strong-viscosity solution firstly for semilinear parabolic partial differential equations and then for path-dependent partial differential equations.

51. Mean-field models and control of multi-agent systems 3 — 16:00 – 17:40

***Chair*: G. Albi, M. Fornasier** — **PrechtlsaalII**

Mean field optimal control

Francesco Solombrino (1), Massimo Fornasier (2),

(1) Chair of Analysis, Munich University of Technology, Munich, Germany (2) Chair of Applied Numerical Analysis, Munich University of Technology, Munich, Germany

We introduce the concept of *mean-field optimal control* which is the rigorous limit process connecting finite dimensional optimal control problems with ODE constraints modeling multi-agent interactions to an infinite dimensional optimal control problem with a constraint given by a PDE of Vlasov-type, governing the dynamics of the probability distribution of interacting agents. While in the classical mean-field theory one studies the behavior of a large number of small individuals *freely interacting* with each other, by simplifying the effect of all the other individuals on any given individual by a single averaged effect, we address the situation where the individuals are actually influenced also by an external *policy maker*, and we propagate its effect for the number N of individuals going to infinity. On the one hand, from a modeling point of view, we take into account also that the policy maker is constrained to act according to optimal strategies promoting its most parsimonious interaction with the group of individuals. This will be realized by considering cost functionals including L^1-norm terms penalizing a broadly distributed control of the group, while promoting its sparsity. On the other hand, from the analysis point of view, and for the sake of generality, we consider broader classes of convex control penalizations. In order to develop this new concept of limit rigorously, we need to carefully combine the classical concept of mean-field limit, connecting the finite dimensional system of ODE describing the dynamics of each individual of the group to the PDE describing the dynamics of the respective probability distribution, with the well-known concept of Γ-convergence to show that optimal strategies for the finite dimensional problems converge to optimal strategies of the infinite dimensional problem.

Mean field games with nonlinear mobilities in pedestrian dynamics

Marie-Therese Wolfram (1), Martin Burger (2), Marco Di Francesco (3), Peter A. Markowich (4),

(1) RICAM, Linz, Austrian Academy of Sciences (2) Institute for Computational and Applied Mathematics, University of Münster, Germany (3) Department of Information Engineering, Computer Sciences and Mathematics, University of L'Aquila, Italy (4) KAUST, Saudia Arabia

In this talk we present an optimal control approach modelling fast exit scenarios in pedestrian crowds. In particular we consider the case of a large human crowd trying to exit a room as fast as possible. The motion of every pedestrian is determined by minimising a cost functional, which depends on his/her position, velocity, exit time and the overall density of people.This microscopic setup leads in the mean-field limit to a parabolic optimal control problem.
We discuss the modelling of the macroscopic optimal control approach and show how the optimal conditions relate to Hughes model for pedestrian flow. Furthermore we provide results on the existence and uniqueness of minimisers and illustrate the behaviour of the model with various numerical results.

Optimal control problems for multi-agent systems

Dante Kalise (1),

(1) Optimization and Optimal Control Group, Radon Institute for Computational and Applied Mathematics, Linz

Multi-agent systems are often introduced in settings where a certain target is sought to be achieved, e.g. flocking, minimum evacuation time, leader tracking, etc. In this talk, we present an optimal control problem formulation of such problems. In this context, multi-agent systems can be steered towards an expected behavior by means of a minimal use of control capabilities. We discuss different problem settings and solution methods. We present numerical experiments assessing the performance of the proposed approximation schemes.

[1] M. Caponigro, M. Fornasier, B. Piccoli, E. Trélat. *Sparse stabilization and control of alignment models*, Mathematical Models and Methods in Applied Sciences, 25(3):521–564, 2015.

[2] M. Bongini, M. Fornasier, D. Kalise. *(Un)conditional consensus emergence under perturbed and decentralized feedback controls* . to appear in Discrete and Continuous Dynamical Systems-A, 2015.

Mean-Field Potryagin Maximum Principle

Mattia Bongini (1), Massimo Fornasier (1), Francesco Rossi (2), Francesco Solombrino (1),

(1) Technische Universität München, Munich, Germany (2) Aix Marseille Université, Marseille, France

The mean-field approach is a powerful tool for giving sense to the notion that a discrete dynamical system converges to a continuous one as the number of agents increases. This technique has been recently used in connection with Γ-convergence to show that a two-populations

discrete optimal control problem converges to an ODE-PDE constrainted optimal control problem: in this model, a discrete population of agents (called “leaders”) interacts with a continuous one (the mass of “followers”), and its target is to steer the entire population towards configurations minimizing a given cost. In this talk we address the problem of deriving optimality conditions for this optimal control problem. We show that these optimality conditions can be seen as the mean-field limit for $N \to \infty$ of the Pontryagin Maximum Principle applied to the two-population discrete system with N followers. This, in turn, enables us a constructive method to derive solutions as limits of solutions of the discrete optimality conditions, which in the end let us establish existence results for these kind of Hamiltonian systems. Finally, we prove that the resulting optimality conditions are indeed Hamiltonian flows in the Wasserstein space of probability measures.

52. Real options 1 — 16:00 – 17:40

***Chair*: P.M. Kort** — **Festsaal**

Anticipative transmission planning under uncertainty

Verena Hagspiel (1), Afzal S. Siddiqui (2,3), Trine K. Boomsma (4),

(1) Department of Industrial Economics and Technology Management, Norwegian University of Science and Technology, Trondheim, Norway (2) Department of Statistical Science, University College London , UK (3) Department of Computer and Systems Sciences Stockholm University, Sweden (4) Department of Mathematical Sciences, University of Copenhagen, Denmark

Transmission system operators (TSOs) build transmission lines to take generation capacity into account. However, their decision is confounded by policies that promote renewable energy technologies. Thus, what should be the size of the transmission line to accommodate subsequent generation expansion? Taking the perspective of a TSO, we use a real options approach not only to determine the optimal timing and sizing of the transmission line but also to explore its effects on generation expansion.

Entry deterrence by timing rather than overinvestment in a strategic real options framework

Nick F.D. Huberts (1), Herbert Dawid (2), Kuno J.M. Huisman (1,3), Peter M. Kort (1,4),

(1) Tilburg University, Tilburg, the Netherlands (2) Bielefeld University, Bielefeld, Germany
(3) ASML, Veldhoven, the Netherlands (4) University of Antwerp, Antwerp, Belgium

This paper considers an incumbent-entrant framework, where the incumbent has the option to extend his current capacity and where the entrant has the option to enter the market by a capacity investment. Our model explicitly considers both optimal timing of the investment and setting the capacity level at the moment of investment. Where in the literature entry deterrence is done by overinvestment, we find instead that entry deterrence takes place by timing: the presence of a potential entrant gives the incumbent the incentive to invest first. The incumbent

only invests a small amount, which is, however, large enough to delay a larger investment by the entrant.

We also consider the situation where the investment decision involves only timing, i.e. the capacity decision is given. In such a case we find that the investment order changes, i.e. now the entrant invests before the incumbent does.

Innovation is considered as an alternative framework where asymmetric firms have the option to make an investment in an innovative product. Both player have the option to leave the old market for the new market.

Capacity choice under uncertainty in a duopoly with endogenous exit

Maria N. Lavrutich (1), Kuno J.M. Huisman (1,2), Peter M. Kort (1,3),

(1) CentER, Tilburg University, Tilburg, The Netherlands (2) ASML Netherlands B.F., Feldhoven, The Netherlands (3) University of Antwerp, Antwerp, Belgium

Applying the real options framework, this article investigates firms' strategic decisions regarding capacity investment in a market with uncertain demand. We formulate a duopoly model where firms become active on the market by making an irreversible investment and henceforth hold an option to exit this market if the demand level falls too low. The combination of three decision components, capacity choice, entry and exit timing, results into multiple trigger strategies for the second investor. In particular, in the presence of a large player in the market it can either choose to coexist with its rival in a duopoly or monopolize the market by installing a sufficiently large capacity. In the endogenous game there exist a preemptive equilibrium where the first investor takes the risk of being forced out of the market upon entry of the second investor. Whether this happens or not depends on the future realizations of the stochastic process.

[1] K.J.M. Huisman, P.M. Kort. Strategic capacity investment under uncertainty. *Forthcoming in the RAND Journal of Economic*, 2015.

[2] P. Murto. Exit in duopoly under uncertainty. *The RAND Journal of Economic*, 35(1): 111–127, 2004.

[3] B.M. Lambrecht. The impact of debt financing on entry and exit in a duopoly. *Review of Financial Studies*, 14(3): 765–804, 2001.

Ambiguity in a real option game

Tobias Hellmann (1), Jacco J.J. Thijssen (2),

(1) Center for Mathematical Economics, Bielefeld University, Germany (2) The York Management School, University of York, United Kingdom

In this paper we study a two-player investment game with a first mover advantage in continuous time with stochastic payoffs, driven by a geometric Brownian motion. One of the players is assumed to be ambiguous with maxmin preferences over a strongly rectangular set of priors. We develop a strategy and equilibrium concept allowing for ambiguity and show that equilibira

can be preemptive (a player invests at a point where investment is Pareto dominated by waiting) or sequential (one player invests *as if* she were the exogenously appointed leader). Following the standard literature, the worst case prior for the ambiguous player if she is the second mover is obtained by setting the lowest possible trend in the set of priors. However, if the ambiguous player is the first mover, then the worst case prior can be given by either the lowest or the highest trend in the set of priors. This novel result shows that "worst case prior" in a setting with geometric Brownian motion and κ-ambiguity does not equate to "lowest trend".

53. Economics of crime — 16:00 – 17:40

***Chair*: G. Feichtinger, A. Seidl, G. Tragler** — **Böcklsaal**

Optimal control of a terror queue

Andrea Seidl (1), Edward H. Kaplan (2), Jonathan P. Caulkins (3), Stefan Wrzaczek (1,4), Gustav Feichtinger (1,5),

(1) ORCOS, Vienna University of Technology, Vienna, Austria (2) Yale School of Management, Yale School of Public Health, and Yale School of Engineering and Applied Science, New Haven, USA (3) Carnegie Mellon University, H. John Heinz III College, Pittsburgh, USA (4) Department of Business Administration, University of Vienna, Vienna, Austria (5) Wittgenstein Centre for Demography and Global Human Capital (IIASA, VID/ÖAW, WU),Vienna Institute of Demography/Austrian Academy of Sciences, Vienna, Austria

The task of covert intelligence agents is to detect and interdict terror plots. [1] treats terror plots as customers and intelligence agents as servers in a queuing model. We extend Kaplan's insight to a dynamic model that analyzes the inter-temporal trade-off between damage caused by terror attacks and prevention costs to address the question of how many agents to optimally assign to such counter-terror measures. We compare scenarios which differ with respect to the extent of the initial terror threat and study the qualitative robustness of the optimal solution. We show that in general, the optimal number of agents is not simply proportional to the number of undetected plots. We also show that while it is sensible to deploy many agents when terrorists are moderately efficient in their ability to mount attacks, relatively few agents should be deployed if terrorists are inefficient (giving agents many opportunities for detection), or if terrorists are highly efficient (in which case agents become relatively ineffective). Furthermore, we analyze the implications of a policy that constraints the number of successful terror attacks to never increase. We find that the inclusion of a constraint preventing one of the state variables to grow leads to a continuum of steady states, some which are much more costly to society than the more forward-looking optimal policy that temporarily allows the number of terror attacks to increase.

[1] Kaplan, E. H. Terror queues. *Operations Research*, 58(4-part-1):773–784, 2010.

Dynamic game of extortion with open loop strategies

Katarzyna A. Kraszewska (1), Agnieszka Wiszniewska-Matyszkiel (2),

(1) Faculty of Mathematics, Informatics and Mechanics, University of Warsaw, Warsaw, Poland (2) Institute of Applied Mathematics and Mechanics, University of Warsaw, Warsaw, Poland

We analyse the dynamic game between local criminal gang and the police concerning extortion from local business owners proposed by Dawid et al. in [1], in which properties of some Markov perfect equilibria were examined.

The extortion level at each time instant is equal to the expected value of loss of capital, resulting from efforts of the gang and the police, in the case of not paying ”protection money”.

Consequently, ”protection money” is always paid, which influences growth of capital stock and can lead to stagnation of the local economy.

The model takes into account also some level of corruption of the police.

We analyse the model with open loop strategies. For this model, we prove a very rare property of equivalence of open loop Nash equilibria with closed loop – Markov perfect Nash equilibria.

[1] H. Dawid, G. Feichtinger and A. Novak. Extortion as an obstacle to economic growth: a dynamic game analysis. *European Journal of Political Economy*, 18:499–516, 2002.

Persistent oscillations in an asymmetric Lanchester model when intelligence depends both on effort and the fraction of insurgents

Andreas J. Novák (1), Gustav Feichtinger (2), George Leitmann (3),

(1) Department of Business Administration, University of Vienna, Austria (2) ORCOS, Vienna University of Technology, Vienna, Austria (3) College of Engineering, University of California, Berkeley, USA

The combat between a regime’s forces and insurgents is modeled in an asymmetric Lanchester-type setting. Since the insurgents are hardly visible for their opponent and ’shots into the dark’ have unpleasant side-effects, the governmental forces have to identify the location and the strength of insurgents. In a simplyfied version where the effort of intelligence is the only control variable and its interaction with the fraction of insurgents on the level of information is modeled in a non-linear way it can be shown that persistent oscillations (stable limit cycles) may be the optimal solution. Moreover, we analyse a more general model, where also the recruitment of governmental troops as well as the attrition rate of the insurgents caused by the regime’s forces, i.e. the ’fist’, are considered as control variables. By using Pontryagin’s maximum principle some interesting insights especially concerning the existence of interior steady state solutions can be obtained.

Extending Lanchester models to a three-player differential game

Jonathan P. Caulkins (1), Gustav Feichtinger (2,3), Dieter Grass (2), Edward H. Kaplan (4), Moshe Kress (5), Andrea Seidl (2), Stefan Wrzaczek (2,6),

(1) Carnegie Mellon University, H. John Heinz III College, Pittsburgh, USA (2) ORCOS, Vienna University of Technology, Vienna, Austria (3) Wittgenstein Centre for Demography and Global Human Capital (IIASA, VID/ÖAW, WU),Vienna Institute of Demography/Austrian Academy of Sciences, Vienna, Austria (4) Yale School of Management, Yale School of Public Health, and Yale School of Engineering and Applied Science, New Haven, USA (5) Operations Research

Department, Naval Postgraduate School, Monterey, USA (6) Department of Business Administration, University of Vienna, Vienna, Austria

Recently, M. Kress proposed to extend the classical Lanchester combat scenario to three opponents to model the war in Syria. Our first aim is to show how the well-known “square-law” of the 2-state Lanchester model translates to the case of three fighters, i.e. how their fighting strengths influence the outcome of the battle. Second, we enrich the pure attrition model on the destruction of enemy forces by state-dependent recruitment flows. And last, but not least, the descriptive approach is extended by adding objectives for the parties, e.g. to eliminate its opponents while surviving at the end. For this a three-person differential game is considered. Interestingly enough, this idea is not only new in the Lanchester context, but it provides also a continuous variant of M. Shubiks early results on truels (i.e. duels between three persons).

Saturday, May 16

54. Plenary — 08:45 – 09:35

***Chair*: Session Chair** — **Festsaal**

Investment dynamics in oligopolistic markets with evolving structure

Herbert Dawid (1),

(1) Bielefeld University, Bielefeld, Germany

This talk provides a survey of recent work analyzing the dynamic strategic interaction between firms competing on markets, in which new submarkets might emerge over time due to product innovations. Firms invest in the build-up of production capacities as well as in knowledge, which is required to successfully carry out product innovations and a main focus of the analysis lies on the interplay between these two types of investment. In particular, it is examined how physical investment is influenced by the present and expected future range of products offered by all firms in the market and how incentives to invest in innovation are affected by the (relative) size of the production capacity of a firm. Piecewise deterministic differential game models are employed to analyze such issues and the analytical as well as numerical approaches used to characterize the properties of Markov-perfect equilibria in these games are reviewed. Finally, it will be shown that in such games multiple locally stable steady states might exist under equilibrium dynamics, giving rise to potential Skiba phenomena. Skiba phenomena in asymmetric differential games of this kind are however associated with conceptual problems, which will also be briefly discussed.

55. Stability and numerical methods for optimal control 2 — 09:55 – 11:35

***Chair*: W. Alt, U. Felgenhauer** — **Prechtlsaal I**

On convergence of the MSE algorithm

Adam Korytowski (1), Maciej Szymkat (2),

(1) AGH University of Science and Technology, Department of Automatics and Biomedical Engineering, Kraków, Poland (2) AGH University of Science and Technology, Department of Applied Computer Science, Kraków, Poland

The method of Monotone Structural Evolution (MSE) is a direct gradient approach to dynamic optimization [1]. Its basic feature is that the controls are defined as concatenations of procedures taken from a given finite set. The selection of a sequence of procedures, their parameters, as well as the switching times are decision variables. The algorithm is divided into periods of gradient optimization in a constant decision space, separated by discrete changes of the space (structural changes). Each structural change produces a new sequence of procedures composing the control, with the preservation of control as a function of time. Structural changes called generations occur when the gradient optimization becomes inefficient, and the Maximum Principle conditions are not satisfied. In principle, they are constructed so as to maximize the efficiency of gradient optimization in the new decision space. This work discusses the questions of convergence of the MSE, following some ideas in [2]. It will be proved that the algorithm stops only at solutions which satisfy the MP conditions, and converges to appropriately defined stationary points.

[1] A. Korytowski and M. Szymkat: Consistent Control Procedures in the Monotone Structural Evolution. Part 1: Theory. *Recent Advances in Optimization and its Applications in Engineering*, ed. by M. Diehl et al., Springer, 247–256, 2010.

[2] H. Axelsson, Y. Wardi, M. Egerstedt, E.I. Verriest: Gradient Descent Approach to Optimal Mode Scheduling in Hybrid Dynamical Systems. *J. Optim. Theory Appl.* 136: 167-186, 2008.

Optimal control problems with discontinuous right hand side

Ekaterina Kostina (1), Olga Kostyukova (2),

(1) University of Marburg, Marburg, Germany (2) Institute of Mathematics, National Academy of Sciences of Belarus, Minsk, Belarus

In this talk we consider optimal control problems with differential equations systems where right hand side may have discontinuities not only in states but also in controls.

Optimal control of systems of differential equations with discontinuous right hand side are widely used to describe numerous applications in natural sciences and engineering, where e.g. there is a necessity to model dynamics with different scales or or with jumps. Moreover, optimal control problems with integer controls may be viewed as a special case of the considered optimal control problems.

We show that application of Filipov rule for such optimal control problems may still lead to incorrect problems and discuss how to reformulate the problems in proper way. Using the reformulated problem we may prove new necessary optimality conditions. Furthermore we analyze how based on the solution of the reformulated problem to construct the approximation to the solution of original problem for any given accuracy.

Switching control based on sparsity

Karl Kunisch (1), Armin Rund (1),

(1) Institute for Mathematics and Scientific Computing, University of Graz, Austria

A novel formulation for switching controls based on convex analysis techniques related to sparse control is introduced. Using a Moreau-Yosida approximation, a family of approximating problems is introduced that is amenable for efficient numerical realization of switching controls using a semismooth Newton method in function space. Numerical experiments for the heat equation and the competitive Lotka-Volterra equations are included.

Bang-bang extremals with multiple switches in the minimum time problem

Laura Poggiolini (1), Marco Spadini (1),

(1) DIMAI, Università degli Studi di Firenze, Italy

We consider the minimum time problem between two given submanifold of the state space for a control-affine dynamics. Assuming there exists an extremal trajectory, i.e. a triplet $\left(\widehat{T},\widehat{\xi},\widehat{u}\right)$ (final time, trajectory, control) satisfying Pontryagin Maximum Principle, and that $\widehat{u}$ is a bang–bang control with a double switch, we give sufficient conditions for such extremal to be state–locally optimal, i.e. optimal according to the following definition:
The trajectory $\widehat{\xi}$ is a state-local minimiser if there are neighborhoods $\mathcal{U}$ of its range $\widehat{\xi}([0,\widehat{T}])$, $\mathcal{U}_0$ of $\widehat{\xi}(0)$ and $\mathcal{U}_f$ of $\widehat{\xi}(\widehat{T})$ such that $\widehat{\xi}$ is a minimiser among the admissible trajectories whose range is in $\mathcal{U}$, with initial point in $N_0 \cap \mathcal{U}_0$ and final point in $N_f \cap \mathcal{U}_f$.
Finally we give sufficient conditions for the optimal control structure to be stable under small perturbations of the data.

All the proofs are carried out by means of Hamiltonian methods and take advantage of invertibility results for PC^1 functions.

56. PDE-constrained optimization 3 — 09:55 – 11:35

***Chair*: I. Neitzel, A. Rösch** — **PrechtlsaalII**

Optimal control of the Fokker-Planck equation

Karl Kunisch (1), Laurent Pfeiffer (1),

(1) Institue of Mathematics and Computer Science, University of Graz, Austria

This talk is devoted to optimal control problems of stochastic differential equations. Instead of minimizing the expectation of a function of the final state, as it is usually done, we minimize a function of the probability density function of the state variable. Such an approach enables in particular to study some risk-adverse problems.

The probability density function of the state variable can be described with the Fokker-Planck equation, a parabolic partial differential equation. Therefore, our approach is close to the approach developped in [1], where a terminal-time tracking objective is used.

Our numerical method is based on a linearization of the problem, for which the usual semi-Lagrangian schemes of the stochastic optimal control theory can be used, in a similar way as in [2].

[1] M. Annunziato and A. Borzì. A Fokker-Planck control framework for multi-dimensional stochastic processes. *Journal of Computational and Applied Mathematics*, 237(1):487–507, 2013.

[2] E. Carlini and F.J. Silva. A Semi-Lagrangian scheme for a degenerate second-order Mean Field Game system. *ArXiv e-prints*. 2014.

Dirichlet control of state-constrained problems

Mariano Mateos (1), Ira Neitzel (2),

(1) Departamento de Matemáticas Universidad de Oviedo, Gijón, Spain (2) Centre for Mathematical Sciences, Technische Universität München, Garching, Germany

PDE constrained optimal control problems with pointwise state constraints are known to cause certain theoretical and numerical difficulties. Some progress has recently been made regarding the numerical analysis of such problems. A priori discretization error estimates and convergence results are available for different classes of problems, including linear-quadratic distributed control problems, problems with Neumann boundary control, or problems with finitely many control parameters. In this talk, we are concerned with a Dirichlet boundary control problem, which admits less regularity for L^2-control functions than for instance Neumann boundary control problems. We will focus on presenting a priori error estimates for the finite element discretization of such linear-quadratic problems with pointwise state constraints in the interior of the domain.

Computational approaches for optimal control problems

Arnd Rösch (1), Gerd Wachsmuth (2),

(1) Fakultät für Mathematik, Universität Duisburg-Essen, Germany (2) Fakultät für Mathematik, Technische Universität Chemnitz, Germany

We study different computational approaches for a linear quadratic optimal control problem with control constraints. Our main focus is on popular methods which does not fit in the well established theory. Usually, such methods are connected with a low computational effort and work well in practice.

In this talk we will give a justification of several approaches. However, some interesting questions will remain open.

A composite step method for the design of a facial implant

Anton Schiela (1), Lars Lubkoll (2), Martin Weiser (2),

(1) Universität Bayreuth, Bayreuth, Germany (2) Zuse-Institut Berlin, Berlin, Germany

We discuss a design problem for a facial bone implant that can be described as an optimal control problem. The main difficulty is the modelling of the facial soft tissue as a nonlinear elastic material.

The resulting optimization problem in function space is tackled by a composite step method. Globalization is achieved by a combination of cubic regularization of the functional and an affine covariant damping strategy for the equality constraints. In the talk we develop the main algorithmic ideas of this method and show numerical examples from applications.

57. Real options 2 — 09:55 – 11:35

***Chair*: P.M. Kort** — **Festsaal**

Firm's optimal decisions of debt renegotiation, asset sale, and liquidation

Michi Nishihara (1), Takashi Shibata (2),

(1) Graduate School of Economics, Osaka University, Japan (2) Graduate School of Social Sciences, Tokyo Metropolitan University, Japan

We consider a situation in which shareholders of a firm in distress have a choice of whether to proceed to liquidation or debt renegotiation at an arbitrary time. We show that a lower volatility and a higher initial coupon increase the shareholders' incentive to choose debt renegotiation to avoid liquidation. When debt renegotiation is optimally chosen, the shareholders decrease the coupon of debt and use equity financing to retire a part of the debt value at the original liquidation time. The shareholders do not prefer partial asset sale in debt renegotiation unless the sale price is higher than the corresponding value of the liquidation case. We also reveal the effects of a high equity financing cost of the firm in distress. A higher equity financing cost reduces the value of debt renegotiation by suppressing the coupon reduction, and then, it increases the shareholders' incentive to liquidate the firm.

Real options and bank bailouts: how uncertainty affects optimal bank bailout policy

Glen Vermeulen (1), Peter M. Kort (1,2),

(1) University of Antwerp, Antwerp, Belgium (2) Tilburg University, Tilburg, The Netherlands

This paper develops a real options consistent bailout decision rule that specifies under which conditions it is optimal to liquidate or bail out a bank based on the amount of liquidity it creates. Due to its construction, the rule incorporates the option value of waiting stemming from the irreversibility of liquidation and bailout decisions and the possibility to delay. We apply the rule to various cases in order to evaluate the quality of bank bailout policy in the EU-15. The main

contribution however lies in the application of real options analysis to the field of bank bailout policy.

Production flexibility and capacity investment: a real-options approach

Xingang Wen (1), Peter M. Kort (1,2), Dolf Talman (1),

(1) CentER, Department of Econometrics & Operations Research, Tilburg University, P.O. Box 90153, 5000 LE Tilburg, The Netherlands (2) Department of Economics, University of Antwerp, Prinsstraat 13, 2000 Antwerp 1, Belgium

This paper considers the investment decision of a firm where it has to decide about the timing and capacity. We allow the firm to produce below the invested capacity. The results show that in a fast growing market, right after the investment, the firm produces below capacity, where the utilization rate (the proportion of capacity that is used for production right after the investment) increases with market uncertainty. On the contrary, the firm produces up to capacity and the utilization rate decreases with market uncertainty in a slowly growing market. In the intermediate, a non-monotonic utilization rate is possible, and the firm produces below capacity right after investment when uncertainty is low and up to capacity when uncertainty is high.

Off-shore enhanced oil recovery in the north sea: how to match co2 demand and supply given uncertain market conditions

Tine Compernolle (1), Kris Welkenhuysen (2), Kuno J.M. Huisman (3,4), Kris Pieters (2), Peter M. Kort (3,5),

(1) Hasselt University, Martelarenlaan 42, 3500 Hasselt, Belgium (2) Royal Belgian Institute of Natural Sciences, Geological Survey of Belgium, Jennerstraat 13, 1000 Brussels, Belgium (3) Tilburg University, Department of Econometrics and Operations Research and Center, Warandelaan 2, 5037 Tilburg, The Netherlands (4) ASML, De Run 6501, 5504 Veldhoven, The Netherlands (5) University of Antwerp, Department of Economics, Prinsstraat 13, 2000 Antwerp, Belgium

CO_2 enhanced oil recovery (CO_2-EOR) entails the injection of CO_2 in mature oil fields in order to mobilize the oil. In particular, the injected CO_2 reduces the oils viscosity and acts as a propellant, resulting in an increased oil extraction rate [1]. Studies regarding the economic evaluation of CO_2-EOR projects consider this project as one investment problem [2, 3], while in reality, both a CO_2 supplier (an electricity producer for whom CO_2 is a by-product) and a CO_2 user (an offshore oil company that exploits oil fields in the North Sea and needs CO_2 for enhanced oil recovery) will have to make investments. Moreover, these studies address uncertainty only by a sensitivity analysis. Given uncertainty in both oil price and CO_2 price under the EU ETS system, we adopt a real options approach [4] and analyze under which economic conditions a CO_2 exchange can be established between an electricity producer and an oil company. We show the impact of price uncertainty on the investment decision of the electricity producer to capture and sell CO_2, and on the decision of the oil company to make the necessary investments to inject CO_2 for enhanced oil recovery. Based on these results, it will

be determined under which economic conditions a CO_2 exchange and transport can take place. Furthermore, also the role of the ETS system will be discussed.

[1] Leach, A., C.F. Mason, and K.v.t. Veld, Co-optimization of enhanced oil recovery and carbon sequestration. *Resource and Energy Economics*, 33(4), 893-912, 2011.

[2] Klokk, et al., Optimizing a CO_2 value chain for the Norwegian Continental Shelf. *Energy Policy*, 38(11), 6604-6614, 2010.

[3] Gaspar Ravagnani, A.T.F.S., E.L. Ligero, and S.B. Suslick, CO2 sequestration through enhanced oil recovery in a mature oil field. *Journal of Petroleum Science and Engineering*, 65, 129-138, 2009.

[4] Dixit, A. and R. Pindyck. *Investment under Uncertainty*. Princeton University Press, 1994.

58. Economic applications of optimal control 2

09:55 – 11:35

***Chair*: S. Chair**

Böcklsaal

Non-constant discounting and the AK growth model

Guiomar Martín-Herrán (1), Francisco Cabo (1), M. Pilar Martínez-García (2),

(1) IMUVa, Universidad de Valladolid, Valladolid, Spain (2) Universidad de Murcia, Murcia, Spain

Recently, there has been a growing literature supporting the hypothesis of consumers whose intertemporal discount rate decreases with the time distance from the present. This paper analyzes the well-known AK endogenous growth model considering individuals with non-constant discounting. The growth rate of the economy is characterized for consumers who cannot precommit their future behaviour. When these consumers act naïvely their optimal policies are time inconsistent and they need to re-optimize continuously. By contrast, time-consistent decisions are made by sophisticated agents who play a game against their future selves. Recently, for naïve consumers and log-utility [1] compares the endogenous AK model under either exponential or hyperbolic discounting. They are observationally equivalent and further, under a plausible assumption (the same present value of a constant infinite stream) the growth rates are identical. We extend this analysis by considering sophisticated agents with iso-elastic utility and prove that the observational equivalence holds true not only for naïve but also for sophisticated consumers. However, under the previous assumption hyperbolic and exponential discounting do not share the same growth rate. Finally, both for naïve and sophisticated consumers we prove that any policy aiming to increase the productivity of the economy is less growth-enhancing than typically predicted by the literature with exponential discounting.

[1] H. Strulik. Hyperbolic discounting and endogenous growth. *Economics letters*, 126:131–134, 2015.

Economics of talent: dynamics and multiplicity of equilibria

Franz Wirl (1), Yuri A. Yegorov (1), Dieter Grass (2), Andrea Seidl (2),

(1) Dept. of Industry, Energy and Environment, University of Vienna, Vienna, Austria (2) ORCOS, Vienna University of Technology, Vienna, Austria

This paper models dynamic interactions between talent and its market. The economics of art and science differs from other branches, by the small role of material inputs and the large role of market accessibility. E.g., an author requires access to (potential) readers as a complement to his productive capital. We study the problem of optimal investments into two stocks: productive capital and access (or bargaining power) that determines how much of the revenues can be accrued. It is shown that there exists a multiplicity of equilibria (under quite general conditions), including interior and boundary (quitting productive activity) outcomes. The separation of the outcomes depends crucially on both idiosyncracies, individual talent and individual market access (including or depending on market size) where the second one differs in particular across geographic, linguistic, and aesthetic dimensions.

Viable economic states for living standards improvement

Jacek B. Krawczyk (1), Wilbur Townsend (1),

(1) Victoria Business School, Victoria University of Wellington, Wellington, New Zealand

Viability theory deals with constrained dynamic systems. It determines a set of initial conditions – the viability kernel – from which a system can be controlled to satisfy some state constraints. In this paper we first provide statistical evidence that the ratio of production factor shares is an important determinant of income inequality. We then develop a state-space model of inequality centred on this ratio. From this we produce viability kernels which guarantee growth while constraining inequality.

On the role of Hartwick's rule in an economy with exhaustible resources

Rudolf Zimka (1), Anton Dekrét (2),

(1) Matej Bel University, Banska Bystrica, Slovakia (2) Cikkerova 11, Banska Bystrica, Slovakia

In models of economies with exhaustible resources the state and price forms of Hartwick's rule and their generalized versions play an important role at finding conditions for intergenerational equity with respect to constant consumptions or to constant value of a utility consumption function. Economists have been trying to make clear the real position of Hartwick's rules in this domain. Especially questions have been studied under which conditions some form of Hartwick's rule is sufficient or necessary for guaranteeing some form of intergenerational equity, or under which conditions the converse of some form of Hartwick's rule with respect to some form of intergenerational equity is possible. In the contribution the price forms of Hartwick's rule are investigated with respect to prices satisfying a special model, which is the cotangent (price) prolongation of the Dasgupta-Heal-Solow-Stiglitz model in the space of price variables. These so

called canonical prices enable: a) to find conditions under which the generalized price Hartwick rule is necessary and sufficient to guarantee the intergenerational equity with respect to price equity (the sum of the price evaluation of the consumption functions change rates is zero at each point of time), b) to find conditions for the converse of the price Hartwick rule with respect to price equity. The notion price equity generalizes the requirement for constant consumption over time.

59. Stability and numerical methods for optimal control 3 — 11:45 – 13:00

***Chair*: W. Alt, U. Felgenhauer** — **Prechtlsaal I**

Discretization and dualization of linear-quadratic control problems with bang-bang solutions

Christopher Schneider (1),

(1) Institute of Mathematics, University of Jena, Germany

We analyze the behavior of solutions for a general class of convex linear-quadratic optimal control problems (LQPs) with control appearing linearly. Assuming that the optimal control is of bang-bang type we show that the solutions are calm functions with respect to perturbation and regularization parameters. The calmness result is applied to prove convergence of the Euler discretization. We then derive the dual problem of the LQP and find that strong duality with zero duality gap holds in this case. These results will be transferred to the discretization of the dual problem. Numerical experiments conclude the talk. We compare the numerical solution of the primal and dual problem, and figure out if computational savings can be obtained.

Convergence results for the discrete regularization of linear-quadratic control problems with bang-bang solutions

Martin Seydenschwanz (1),

(1) Siemens AG – Research and Technology Center, Munich, Germany

We analyze a combined regularization-discretization approach for a class of linear-quadratic optimal control problems. By choosing the regularization parameter α with respect to the mesh size h of the discretization we approximate the optimal bang-bang control. Under weaker assumptions on the structure of the switching function we generalize existing convergence results from [1], [2] and prove error estimates of order $\mathcal{O}(h^{1/(k+1)})$ with respect to the controllability index k. The theoretical results are strongly connected to the findings in [3], [4] and will be illustrated by numerical examples.

[1] W. Alt, R. Baier, M. Gerdts and F. Lempio: *Error bounds for Euler approximation of linear-quadratic control problems with bang-bang solutions*, Numerical Algebra, Control and Optimization **2** (2012), 547–570.

[2] W. Alt and M. Seydenschwanz: *Regularization and discretization of linear-quadratic control problems*. Control and Cybernetics, **40**(4), (2011), 903–921.

[3] J. Haunschmied, A. Pietrus and V.M. Veliov: *The Euler method for linear control systems revisited*. In: I. Lirkov, S. Margenov and J. Wasniewski (eds.): Large-scale scientific computing – LSSC 2013, Lecture Notes in Computer Science, **8353**, (2014), 90–97.

[4] M. Quincampoix and V.M. Veliov: *Metric regularity and stability of optimal control problems for linear systems*. SIAM J. Control Optim., **51**(5), (2013), 4118–4137.

Program packages method for solving closed-loop guidance problem with incomplete information for linear systems

Nikita V. Strelkovskii (1,2),

(1) IIASA, Laxenburg, Austria (2) Lomonosov Moscow State University, Russia

The method of program packages [1] is a tool for identyfying the solvability conditions of guaranteed positional control problems when information on observed states is incomplete. In this talk a version of the method applicable to the problem of guaranteed positional guidance of a linear control system to a convex target set at a specified time is presented. The observed signal on the system's states is assumed to be linear and the set of its admissible initial states is assumed to be finite. The method is based on clusterisation of the set of initial states according to the corresponding homogeneous signals and the moments of their separation from each other. A program package is a set of programs which are parametrized by the initial states and satisfying the "non-anticipatory" condition. It is proved that the problem of guaranteed positional guidance is equivalent to the problem on the program package guidance which itself is equivalent to the program guidance of an extended linear control system to an extended convex target set [2]. For the latter problem a solvability criterion which reduces the task to the solution of a finite-dimensional optimization problem is produced using the separation theorem for convex sets. A procedure of construction the guiding program package and the corresponding guiding positional strategy which solves the given problem is described.

[1] A. V. Kryazhimskiy and Yu. S. Osipov. Idealized program packages and problems of positional control with incomplete information. *Trudy Inst. Mat. i Mekh. UrO RAN*, 15(3): 139–157, 2009.

[2] A. V. Kryazhimskiy and N. V. Strelkovskiy. An open-loop criterion for the solvability of a closed-loop guidance problem with incomplete information. Linear control systems. *Trudy Inst. Mat. i Mekh. UrO RAN*, 20(3): 132–147, 2014.

60. Singularities and optimality conditions in control theory — 11:45 – 13:00

***Chair*: A. Zaslavski** — **PrechtlsaalII**

On average control generating families related to problems of control of singularly perturbed systems

Vladimir Gaitsgory (1),

(1) Department of Mathematics, Macquarie University, Sydney, Australia

It is known that, under certain conditions, the dynamics of the slow components of a singularly perturbed (SP) control system is approximated by solutions of the averaged system, in which the role of controls is played by measure-valued functions. A family of controls and the corresponding solutions of the fast subsystem is called average control generating (ACG) if it generates a state-control trajectory of the averaged system.

We will state sufficient and necessary conditions of optimality of ACG families in problems of optimal control considered on the solutions of the averaged system, and we will discuss a linear programming based approach to numerical construction of near optimal ACG families. The theoretical results will be illustrated with numerical examples.

[1] V. Gaitsgory and S. Rossomakhine. Averaging and Linear Programming in Some Singularly Perturbed Problems of Optimal Control. *Journal of Applied Mathematics and Optimization*, Published online in June 2014, DOI 10.1007/s00245-014-9257-1.

[2] V. Gaitsgory, S. Rossomakhine and L. Manic. On average control generating families for singularly perturbed optimal control problems with long run average optimality criteria *Set-Valued and Variational Analysis*. Published online in October 2014, DOI 10.1007/s11228-014-0306-3.

Discrete singularly perturbed optimal control problems

Galina A. Kurina (1), Mikhail G. Dmitriev (2), Desineni S. Naidu (3),

(1) Voronezh State University, Voronezh, Russia (2) Russian State Social University, Moscow, Russia (3) University of Minnesota Duluth, Duluth, USA

There are reviews of publications for the last years devoted to singularly perturbed optimal control problems, e.g. [1], [2], discrete problems in which occupy a negligible part. We present the review of various types of discrete singularly perturbed optimal control problems and methods for solving them. Namely, we give the review of the publications where asymptotic solutions of discrete optimal control problems have both the form of expansions of boundary layer type and regular expansions with respect to non-negative powers of a small parameter. Methods of decoupling motions into pure-slow and fast ones are considered also. Besides, we survey publications devoted to the stabilization of discrete systems, control problems by discrete systems with a small step, descriptor and stochastic systems, and game problems.

[1] M.G. Dmitriev and G.A. Kurina. Singular perturbations in control problems. *Automation and Remote Control*, 67(1):1–43, 2006.

[2] Yan Zhang, D.S. Naidu, Chenxiao Cai, and Yun Zou, Singular perturbation and time scales in control theories and applications. *Int. J. Information and Systems Sciences*, 9(1):1–36, 2014.

A non smooth Chow-Rashevsky theorem

Monica Motta (1), Franco Rampazzo (1),

(1) Dipartimento di Matematica, Universita di Padova, Padova, Italy

Chow-Rashevsky theorem (also known as Hormander condition) is in fact a sufficient condition for local controllability of a family F of vector fields. It involves a rank condition concerning the vector fields of F and their iterated Lie brackets. In its original form, it is formulated for C^∞ vector fields, but its validity has been proved for less stringent assumptions. However, most of the available result assume that the brackets involved in the rank test are single-valued continuous functions. This of course is not the case for the Lie bracket $[f, g] := \nabla g f - \nabla f g$ of two Lipschitz continuous vector fields f, g. By means of a notion of set-valued Lie brackets valid for cases where the usual derivative might not exist (as in the previous example), we give a a result of local controllability extending in various ways the classical Chow-Rashevsky theorem. Applications to Control Theory and first and second order PDE's will be mentioned as well.

61. Non-monotonic dynamics in economics — 11:45 – 13:00

***Chair*: M.C. Camacho, C. Saglam** — **Festsaal**

Spatial externality and indeterminacy

Emmanuelle Augeraud-Véron (1), Arnaud Ducrot (2),

(1) MIA, University of La Rochelle, France (2) IMB, University of Bordeaux, France

We present a theoretical one-sector growth model with increasing returns (such as in Benhabib and Farmer [1]) which takes into account technological dependences among countries that are modeled by spatial externalities. These spatial externalities play a role in the process of economic growth. Many empirical studies have shown the importance of these spillovers on economic growth, both at a national level (Easterly and Levine [2] and at a regional level. According to Quah [3], "spatial spillover factors matter more than national, macro ones". The dynamics we consider can be written as a system of structured algebraic-differential equations. We study the impact of these externalities on existence and uniqueness of a local solution in the neighbourhood of the homogeneous steady state ([4]). We also characterize indeterminacy criteria for this spatial model, and give a definition of degree of indeterminacy appropriate for structured models. We then consider the impact of spatial externalities on indeterminacy taking into account symmetric and asymmetric relationships among neighbours.

[1] Benhabib, J. and Farmer, R. E. A., 1994. Indeterminacy and Increasing Returns. *Journal of Economic Theory*, 63(1), 19–41.

[2] Easterly, W. and Levine, R. (2001). It's not factor accumulation: stylized facts and growth models. *World Bank Economic Review*, 15, 177–219.

[3] Quah, D. T., 1996. Regional convergence clusters across Europe. *European Economic Review*, 40, 951–958.

[4] Camacho, C. and Zou, B., 2004. The spatial Solow model. *Economics Bulletin*, 18(2), 1–11.

Do incentives in free trade induce free-riding?

Zahra Abdi (1), Hamideh Esfahani (1), Mehdi Fadaee (1),

(1) Institute for Management and Planning Studies, Tehran, Iran

We study a two country world economy where there is one firm in each country producing the same homogeneous good. Production process of firm in brown country generates emissions that add to a stock of transboundary CO2 pollution which evolves over time. The green country whose firm has clean technology reduces CO2 concentration through the carbon capture and storage process. Using a dynamic model we show that despite having a firm with cost advantage, bilateral trade cannot improve brown country's welfare for relatively large cost of storage. Levying environmental tariff, green country's government can induce polluting foreign firm to adopt a clean technology even if the storage cost is relatively small.

On some simple dynamics for vintage growth with heterogenous capital goods

Jean-Pierre Drugeon (1), Hippolyte d'Albis (2),

(1) Paris School of Economics and Centre National de la Recherche Scientifique (2) Paris School of Economics and University of Paris 1 Panthéon-Sorbonne

This article analyses the role of a heterogenous goods structure for the production set—distinct technologies for the two investment good and the mixed good—of an economy where the laws of motion of the capital stocks build from *recent* but also *old* investment. The purpose of the current contribution is then to illustrate how the consideration of a heterogeneous industries can enrich the understanding of the long run in economies with complex aging and generations structures. Of particular relevance is an extension with two capital goods where the physical capital good has a ageing and vintage structure that is distinct from the human capital one. The basic approach proceeds the decentralised two-sector optimal growth model of Uzawa [1962] agmented by vintage capital structures that result in distinct augmented laws of accumulation for the fraction of the capital good that are employed in the capital good industry and the consumption good industry. Solving the problems of the firms, it appears that both are of dynamic nature: while the labour factor is shared according to classical static rules, the allocation of investment proceeds from a dynamic allocation problem on top of the static allocation one between the two industries. Even though it is not any longer possible to let the analysis rest upon a classic and atemporal production possibility frontier, the analysis completes a careful examination of the interior solutions that results in a vintage and time-dependent augmented production possibility frontier. It then becomes possible to examine the main facets of an inter temporal equilibrium with an endogenously determined rate of survival for the capital stock. The augmented form of the Euler equation builds from the echo effects associated with the extra forward and backward dimensions consubstantial to the vintage capital. The long-run definition of the capital stock is enriched through these formulation and allows for comparative statics augmented by the

endogenously determined survival rate of sectoral past investments. It is shown that this survival rate emerges as a weighted average of the parameters that describe the survival rates of investment within any of the industries. An appraisal of the dynamical properties of this environment reveals the emergence of intertwined echo effects for the investments: interestingly, even though it remains possible to derive conditions for long-run convergence, this can only proceed through oscillating sequences for the capital stock whose feature directly result for the nature of the allocation process between the two industries.

A dynamic general equilibrium IS-LM model

Takashi Kamihigashi (1),

(1) RIEB, Kobe University, Kobe, Japan

We study a simple model that unifies the Ramsey model, Kurz's growth model with wealth effects, and Sidrauski's money-in-the-utility-function model. In a very special case, the dynamics of the model are characterized by two curves that resemble the traditional IS and LM curves. This special case has features such as deflationary spirals and a liquidity trap. Thus our model unifies neoclassical growth theory and IS-LM analysis in a simple manner.

62. Differential games in marketing and operations — 11:45 – 13:00

***Chair*: G. Zaccour** — **Böcklsaal**

The role of quality strategies in the interface between conformance quality and goodwill dynamics

Pietro De Giovanni (1), Gila Fruchter (2),

(1) Department of Operations Management, ESSEC Business School, Paris, France (2) Graduate School of Business Administration, Bar-Ilan University, Ramat-Gan, Israel

In this paper, we investigate the effect of appraisal and prevention strategies as well as internal and external failures in both the conformance quality and the goodwill dynamics. Conformance quality represents the capability to make free-defect products and is accumulated over time through investments in appraisal and prevention. Prevention costs involve the costs of planning and designing the production process to ensure high conformance quality standards. Appraisal costs include all costs for inspecting, testing and supervising both the materials and the finished products. Instead, cost for failures are all costs incurred because poor conformance quality exists. We investigate the trade-offs between all costs for quality policy as well as their impact on the goodwill dynamics with the purpose to identify the optimal quality strategies that lead to high profits, low cost for quality, and low unsatisfied customers.

Defensive, offensive, and generic advertising in a Lanchester model with market growth

Steffen Jørgensen (1), Simon Pierre Sigué (2),

(1) Department of Business and Economics, University of Southern Denmark, Odense, Denmark
(2) Faculty of Business, Athabasca University, Canada

The paper considers a duopolistic market in which firms compete over time. We focus on advertising competition where each firm may use three types of advertising: offensive advertising which has the purpose of attracting customers from the rival firm, defensive advertising which aims at protecting a firm's customer base from the competitors' attacks, and generic advertising the purpose of which is to enhance industry sales. The paper addresses questions like: How should a strategy, for each of the three types of advertising effort, be designed? How would the corresponding time paths of sales look like? We suggest a finite horizon differential game model to answer these questions and provide closed-form expressions for equilibrium advertising strategies and sales rate trajectories. It turns out that advertising strategies can be expressed nicely in terms of the shadow prices (and their sum and their difference) of the sales rates of the firms.

Quality effects in different advertising models - an impulse control approach

Puduru V. Reddy (1), Stefan Wrzaczek (2), Georges Zaccour (1),

(1) GERAD, HEC Montréal, Canada (2) Vienna University of Technology, Austria

In this paper, we integrate quality as a control variable in the three classical dynamic optimal control models of advertising, namely, Nerlove-Arrow (1962), Vidale-Wolfe (1957) and Ozga (1960) models. We assume that decisions in quality improvement can only be made at some exogenously given instants of time, and consequently we use the formalism of hybrid optimal control to determine optimal advertising and quality investments. We report numerical results for the three models and discuss the impact of adding quality on the results.

Poster presentations

Optimization of second order discrete approximation inclusions

Gulseren Çiçek (1), Elimhan N. Mahmudov (2),

(1) Istanbul University, Faculty of Science, Dep. of Mathematics, Vezneciler, Istanbul, Turkey
(2) Industrial Engineering Department Faculty of Management, Istanbul Technical University 34367 Maçka Istanbul, Turkey

In the present paper we consider the so-called Generalized Cauchy Problem for Second Order Differential Inclusions. We find out the approximation conditions for this problem. Locally adjoint mapping is our basic tool to formulate necessary and sufficient conditions for the optimality of the discrete approximation problem. Then by passing to the limit, sufficient optimality conditions to the optimal problem described by differential inclusions are established.

Approximation and optimization of third order polyhedral discrete and differential inclusions

Sevilay Demir (1), Elimhan N. Mahmudov (2),

(1) Istanbul University, Faculty of Science, Dep. of Mathematics, Istanbul, Turkey (2) Industrial Engineering Department Faculty of Management, Istanbul Technical University 34367 Maçka Istanbul, Turkey

The paper concerns with the necessary and sufficient conditions of optimality third order polyhedral optimization described by polyhedral discrete (P_D) and differential (P_C) inclusions. By converting the problem (P_D) into the a problem with geometric constraints, we are able to obtain the conditions of optimality for third order polyhedral discrete inclusions in term of the Euler-Lagrange polyhedral discrete inclusions and special formulated the transversality conditions. Then the necessary and sufficient conditions of optimality for discrete-approximation problem (P_{DA}) are formulated using the transversality condition and approximation method for the continuous polyhedral problem (P_C) governed by polyhedral discrete inclusions. By virtue of the obtained results, we prove the sufficient conditions of optimality for the problem (P_C) . It turns out that the concerned method requires some special equivalence theorem, which allow us to make a bridge between (P_D) and (P_C) problems.

Optimal pricing of an improving durable good with endogenous demand

Serhat Gezer (1,2),

(1) Chair for Economic Theory and Computational Economics, Bielefeld University, Bielefeld, Germany (2) Bielefeld Graduate School of Economics and Management, Bielefeld University, Bielefeld, Germany

We investigate optimal pricing decisions of a monopolist in a discrete dynamic framework where demand is modeled endogenously by actions of consumers and new consumer arrive each period. The firm sells an improving durable good which will be improved in the future and consumers are expecting that. We obtain several optimal price paths which might exhibit periodic clearance sales. We characterize two consumer segments whose behaviours are fundamentally different and derive implications for the monopolist.

Optimal double control problem for a PDE model of goodwill dynamics

Mariusz Górajski (1), Dominika Bogusz (1),

(1) Department of Econometrics, University of Łódź, Łódź, Poland

We propose a new optimal model of product goodwill in a segmented market where the state variable is described by a partial differential equation of the Lotka–Sharp–McKendrick type (cf. [1]). In order to maximize the sum of discounted profits over a finite time horizon, we control the advertising efforts which influence the state equation and the boundary condition. Moreover,

we introduce the mathematical representation of consumer recommendations in a segmented market. Based on the semigroup approach, we prove the existence and uniqueness of optimal controls. Finally, using a maximum principle from [2], we propose a numerical algorithm to find the optimal solution.

The product goodwill G is described by PDE of the form:

$$\begin{cases} \frac{\partial G(t,a)}{\partial t} + \frac{\partial G(t,a)}{\partial a} + \delta(a)G(t,a) = u^{\rho}(t,a) & (t,a) \in [0,T] \times [0,1]\,, \\ G(t,0) = \int_0^1 \left(R(a)G(t,a) + u^{\rho}(t,a)\right) da + u_0^{\rho}(t) & t \in [0,T]\,, \\ G(0,a) = G_0(a) & a \in [0,1]\,. \end{cases} \tag{1}$$

Here, $G(t,a) \geq 0$ is the product goodwill at time t for consumer segment a, where $a \in [0,1)$ equates with the consumer usage experience, $u(t,a), u_0(t) \geq 0$ are the advertising efforts at time t directed to consumer segment a and to new consumers, respectively, $\rho \in (0,1]$, $R(a) \geq 0$ is the rate of consumer recommendation for consumers with usage experience a and $\delta(a) \geq 0$ is the depreciation rate of the product goodwill in consumer segment a.

[1] D. Bogusz, M. Górajski. Optimal double control problem for a PDE model of goodwill dynamics. *arXiv:1411.0880 [math.OC]*, 2014.

[2] G. Feichtinger, G. Tragler, V. Veliov. Optimality conditions for age-structured control systems. *Journal of Mathematical Analysis and Applications*, 288 (1), 47–68, 2003.

A Stackelberg oligopoly with sticky prices and myopic follower

Katarzyna Kańska (1), Agnieszka Wiszniewska-Matyszkiel (2),

(1) Faculty of Mathematics, Informatics and Mechanics, University of Warsaw, Poland (2) Institute of Applied Mathematics and Mechanics, University of Warsaw, Poland

We study a model of a dynamic duopoly of Stackelberg type. Prices which we consider are sticky and the leader is fully conscious about their behaviour, while the follower is myopic.

The model is designed to cope with situations in which the initial price is below the static Stackelberg price and it explains why price can remain below this level also at steady state.

We study both situations in which the leader uses open loop and feedback strategies and calculate resulting equilbria of the considered model.

We derive the complete description of the equilibrium strategies of both players and the equilibrium price trajectory, not only steady state values of these functions.

One of interesting results is that the production of the leader in the open loop case is not greater than his production at feedback equilibrium (with strict inequality oustside some small initial interval), with opposite relation for the follower.

Another one is monotonicity of those strategies. Equilibrium strategies of the leader are increasing over time, while strategies of the follower are decreasing, and the inequalities are strict beside some small initial intervals. Moreover, strategies of both players intersect – firstly production of the leader is smaller than production of the follower, or even it can be 0, then it becomes larger.

We obtain also interesting asymptotic properties. Among other things, as the speed of adjustment of price tends to infinity, which corresponds to immediate price adjustment, only the open

loop equilirium price and production levels of both players converge to their static Stackelberg duopoly levels.

Multidimensional model of capital accumulation with vintage structure

Marta Kornafel (1),

(1) Department of Mathematics, Cracow University of Economics, Poland

The model of capital accumulation with n goods modelled by initial-boudary problem with transport equation will be presented. In the considerations the ageing process of capital goods may experience some discontinuities. The existence, uniqueness of viscosity solutions and their stability with respect to perturbations of coefficients will be studied. The result on cost minimization problem will be provided as well.

A new approach to the minimum time problem and its numerical approximation

Thuy T.T. Le (1), Lars Grüne (2),

(1) University of Padova, Italy (2) University of Bayreuth, Germany

We introduce a new formulation of the minimum time problem in which we employ the *signed* minimum time function positive outside of the target, negative in its interior and zero on its boundary. Under some standard assumptions, we prove the so called *Bridge Dynamic Programming Principle* (BDPP) which is a relation between the value functions defined on the complement of the target and in its interior. Then, owing to BDPP, we obtain the error estimates of a semi-Lagrangian discretization of the resulting Hamilton-Jacobi-Bellman equation. In the end, we provide numerical tests and error comparisons which show that the new approach can lead to significantly reduced numerical errors.

Uniqueness of limit cycles for a limiting case of the chemostat: does it justify the use of logistic growth rates?

Torsten A. Lindström (1), Yuanji Cheng (2),

(1) Department of Mathematics, Linnaeus University, Sweden (2) School of Technology, Malmö University, Sweden

In this paper we discuss commonly used approximations that may lead to non-removable dependency terms potentially affecting the long run qualitative behavior of the involved equations. We prove that these terms do not produce such effects in the simplest and most interesting biological case, but the general case is left open.

Our main result is a rather complete analysis of an important limiting case. Once complete knowledge of the qualitative properties of simple models is obtained, it greatly facilitates further studies of more complex models. A consequence of our analysis is that standard methods can be applied. However, the application of those methods require non-trivial estimates in order to make them valid for all values of the parameters.

Optimization of fourth order discrete and differential inclusions

Elimhan N. Mahmudov (1,2),

(1) Industrial Engineering Department Faculty of Management, Istanbul Technical University, Turkey (2) Azerbaijan National Academy of Sciences, Institute of Control systems, Azerbaijan

The paper concerns the necessary and sufficient conditions of optimality for Cauchy problem of fourth order discrete and differential inclusions. The main problem is formulation of the fourth order adjoint discrete and differential inclusions and transversality conditions, which are peculiar to problems including fourth order derivatives and approximate derivatives. Thus the necessary and sufficient conditions of optimality are obtained incorporating the Euler-Lagrange and Hamiltonian forms of inclusions. Derivation of optimality conditions are based on the apparatus of locally adjoint mapping (LAM). Moreover in the application of these results we consider the fourth order linear discrete and discrete-approximate inclusions.
In this paper we deal with the problem for fourth order differential inclusions

$$\begin{aligned} \text{minimize} \quad & J[x(\cdot)] = \int_0^1 g(x(t),t)dt + \varphi_0(x(1),x'(1),x''(1),x'''(1)) \quad (1)\\ & x^{(IV)}(t) \in F(x(t),x'(t),x''(t),x'''(t)) \text{ , a.e. } \quad t \in [0,1], \quad (2)\\ & x(0)=\alpha_0,\ x'(0)=\alpha_1,\ x''(0)=\alpha_2,\ x'''(0)=\alpha_3. \quad (3) \end{aligned}$$

Here $F : \mathbb{R}^{4n} \to P(\mathbb{R}^n)$ is a set-valued mapping, $g(\cdot,t) : \mathbb{R}^n \to \mathbb{R}^1$ is continuous with respect to x, $\varphi : \mathbb{R}^{4n} \to \mathbb{R}^1$ proper function and $\alpha_k, k = 0,1,2,3$ are fixed vectors. The problem is to find an arc $\widetilde{x}(t)$ of the Cauchy problem $(1)-(3)$ for the fourth order differential inclusions satisfying (2) almost everywhere (a.e.) on $[0,1]$ and the initial conditions (3) that minimizes the Bolza functional $J[x(\cdot)]$. Here, a feasible trajectory $x(\cdot)$ is an absolutely continuous function on a time interval $[0,1]$ together with the third order derivatives for which $x^{(IV)}(\cdot) \in L_1^n([0,1])$. For such problems we derive necessary and sufficient conditions of optimality.

Optimal Bayesian control chart for the proportion of defectives

Viliam Makis (1), Leila Jafari (1), Farnoosh Naderkhani (1),

(1) Department of Mechanical and Industrial Engineering, University of Toronto, Toronto, Canada

In this paper, we develop an optimal Bayesian scheme to control the proportion of defective items produced over a long production run. Traditional p- chart has been widely used for this purpose in industries for many years. It is well known that traditional non-Bayesian charts are not optimal, but very few optimal Bayesian control charts have been developed in the literature, mostly considering finite horizon. We formulate the problem of controlling the proportion of defective items produced in the optimal stopping framework. The objective is to determine the optimal stopping rule minimizing the long-run expected average cost per unit time considering partial information obtained from process monitoring. It is proved that under standard operating and cost assumptions the control limit policy is optimal. An algorithm is presented to find the optimal control limit and the corresponding average cost, illustrated by a numerical example. A comparison of the developed optimal Bayesian control chart with the traditional p-chart is also provided.

Computation and visualization of control systems reachable sets with parallel processing on Graphics Processing Units

Alina O. Novikova (1),

(1) Lomonosov Moscow State University, Russia

The solution of a wide range of optimal control problems involves constructing reachable sets and their analogs. The pixel method of constructive description and determination of the reachable sets for a given controlled system is described and implemented. For the initial set we build the characteristic matrix in the following way: all the set is covered up with the grid. The size of this grid depends on our implementation. If the element from the initial set reaches the grid node, the corresponding position of the characteristic matrix is 1, otherwise it is 0. Then at every step we work with characteristic matrix. We go from one moment of time to another with time steps and we solve the Cauchy problem every time.

In the pixel method parallel computing is applied to characteristic matrix work, because the work with every pixel element goes independently in parallel way. Presented algorythm is used to solve the problem of construction of reachable sets numerically. CUDA (Compute Unified Device Architecture) technology allows to use Graphics Processing for parallel processing Units. CUDA is a complex of software and hardware tools. CUDA is a parallel computing platform and a programming model. It dramatically increases the computing performance by increasing the power of the graphics processing units (GPU).

The effectiveness of the approach is demonstrated on several examples. Based on realistic numerical values for control examples we conclude that the computed reachable sets are in fact full thrust. The approach may be applied to multidimensional problems.

Optimal control problem for two-sector economic model with Cobb–Douglas production function

Mikhail V. Orlov (1), Yury N. Kiselev (1), Sergey M. Orlov (1),

(1) Applied Mathematics Faculty, Lomonosov Moscow State University, Russia

A few resource allocation problems in a two-sector model with the Cobb–Douglas production function and different functional costs are considered. The planning horizon may be finite or infinite. A constructive description of the optimal solution is proposed in each case. The problems solution is based on the Pontryagin maximum principle. The optimal solution may have a singular arc. In all cases the correspondent singular arc is described by an analytical way. Moreover, the singular trajectory is the same in all problems. The optimality of an extreme solution is proved using the theorem on the sufficient optimality conditions in terms of constructions of the maximum principles. One of the studied problems with different production functions is open to biological interpretation under the model of balanced growth of plants within a given limited time span.

A thorough analysis of dynamic game oligopoly model with sticky prices

Agnieszka Wiszniewska-Matyszkiel (1), Marek Bodnar (1), Fryderyk Mirota (2),

(1) Faculty of Mathematics, Informatics and Mechanics, University of Warsaw, Poland (2) Institute of Applied Mathematics and Mechanics, University of Warsaw, Poland

In this paper we present an extensive analysis of a dynamic model of oligopoly with sticky prices.

We calculate both symmetric feedback Nash equilibria and symmetric open loop Nash equilibria and compare resulting trajectories of price and production.

Although our paper is not the first such model of oligopoly, it appears to be the first paper in which non-stationary open loop Nash equilibria were rigorously calculated.

Therefore, our analysis allows us to study and compare closed loop and open loop Nash equilibria which are not constant over time.

We prove that feedback equilibrium production is always greater or equal to the open loop equilibrium production, and strictly greater from some time instant on, with reverse inequality for prices.

Moreover, equilibria are only piecewise differentiable.

We also analyse behaviour of equilibrium price, production and aggregate production as functions of parameters of the model and we obtain, among others, monotone convergence of steady states of open loop Nash equilibrium price and production to analogous levels of the static Cournot-Nash equilibrium as the speed of adjustment tends to infinity, while for feedback convergence is to some nontrivial convex combination of the static Cournot-Nash equilibrium and the competitive equilibrium.

[1] A. Wiszniewska-Matyszkiel, M. Bodnar, F. Mirota. Dynamic Oligopoly with Sticky Prices: Off-Steady-State Analysis. *Dynamic Games and Application*, doi: 10.1007/s13235-014-0125-z, 2014.

Sport economics of talent: individual performance, competition and reward

Yuri A. Yegorov (1),

(1) Dept. of Industry, Energy and Environment, University of Vienna, Vienna, Austria

This paper models dynamic interactions between talent, training and rewards in sports. The economics of sports differs from other markets by bias in reward structure. Very often “the winner takes it all”, while efforts are produced by many. The paper starts from the discussion of a concept of talent as individual heterogeneity in abilities that can create result only after investment in training. Then we have dynamic optimization problem for an individual talent, with an optimal path of accumulating sportive capital. However, without rivals there is no insentive for training. Thus only game can be studied, and we consider its simple version with 2 rivals and 2 periods. An equilibrium is also possible for heterogeneity in skills due to random factors that influence outcome. The distribution of individual talents and market structure also has an influence on the reward.

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